Spatially Balanced Sampling Methods in Household Surveys

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Abstract

Household surveys are the most common type of survey used for providing information about the social and economic characteristics of a population of people. In these surveys, information is usually collected by sampling the houses where people live and then enumerating one or more persons at each home. Current sampling methodologies used in designing household surveys generally do not take into account the spatial structure of populations. This may lead to selection of units (i.e., households, individuals) near to each other that usually provide similar information in the sample. As a result, the selected sample tends to be less efficient than a sample that reflects all attributes of the population.

Spatially balanced sampling is a popular design for selecting samples from natural resources and environmental studies, which avoids selecting neighbouring units in the same sample. Spatially balanced sampling design ensures the selection of a representative sample by providing a spatial coverage of a region corresponding to the population of interest.

This doctoral thesis aims to assess the possibility of applying spatially balanced sampling in designing household surveys. After investigating spatially balanced methods available in the literature, balanced acceptance sampling (BAS), developed by Robertson et al (2013) is considered for further investigation in this study.

This research comprises two main parts: (1) exploring the characteristics of BAS from a practical perspective, (2) promoting the application of spatially balanced sampling in household surveys. The first part looks into the advantages of the BAS method in practical cases. It aims to highlight the potential advantages of the BAS method for selecting samples in practical situations in environmental studies. The flexible characteristics of BAS and its practical benefits (e.g., being able to accommodate missed sampling units and the ability to add extra sampling units during survey implementation) discussed in the first part, show that BAS has the potential to be extended for application in other surveys, specifically, household surveys.

In the second part, the applicability of spatially balanced sampling in household surveys is assessed. A technique for selecting a spatially balanced sample from a

discrete population, called BAS-Frame, is introduced. The spatial and statistical properties of the proposed method are investigated through conducting simulation studies using the census 2013 meshblocks of selected regions in New Zealand. The results from these simulation studies show that the proposed method is sufficiently robust in spreading the sample over the population of interest. In addition, it is seen that applying spatially balanced sampling in selecting samples for household surveys provides more precise estimates when compared to non-spatially balanced sampling methods.

The feasibility of spatially balanced sampling methods to deal with some practical aspects of designing a household survey is also investigated in the second part (e.g., designing a primary sampling unit (PSU) which meet a pre-specified minimum number of sampling units, designing longitudinal surveys, and selecting a sample in the presence of auxiliary variables). A method on the basis of the BAS-Frame is developed to merge undersized units with their nearby units as much as possible to define PSUs. A simulation study shows that the proposed method is more powerful than the conventional method (i.e., the Kish method) in combining the undersized units with their undersized neighbours. The application of the BAS-Frame for controlling overlap between rotation groups in the longitudinal designs is discussed. Finally the performance of the BAS-Frame in spreading the sample over the space of the auxiliary variables available in the frame is investigated. This study shows that in the case of the existence of a small number of auxiliary variables (fewer than five variables), the BAS-Frame can provide a good spread, not only over the geographical space of the population, but also over the space of the auxiliary variables.

This research, by studying multiple concepts of spatially balanced sampling, leads to better understanding of these sampling methods, and the advantages of extending their applications to household surveys

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List of contents

Abstract i

Acknowledgements iii

Chapter 1 Introduction 1

1.1 Background 1

1.2 Research Motivations 2

1.3 Research Objectives and Scope of Work 4

1.4 Organization of Thesis 5

1.5 References 7

Chapter 2 Sampling Design Approaches 9

2.1 Introduction 9

2.2 Essential Concepts and Notations 10

2.3 Probability Sampling Design 11

2.4 Review on Some Classic Sampling Designs 12

2.4.1 Simple Random Sampling 13

2.4.2 Stratified Sampling 14

2.4.3 Cluster and Multistage Sampling 15

2.5 Sampling Designs in Household Surveys 16

2.5.1 Defining PSUs in Household Surveys 17

2.5.2 Stratifying PSUs in Household Surveys 19

2.5.3 Sampling PSUs in Household Surveys 19

2.5.4 Longitudinal Designs in Household Surveys 19

2.6 Spatial Autocorrelation and *Moran's I* Index 20

2.6.1 Review of Some Spatially Balanced Sampling Methods 22

2.6.2 Parameter Estimation in Spatially Balanced Sampling Methods 32

2.6.3 Spatial Coverage 33
2.7 Conclusions 36
2.8 References 36
Chapter 3 Balanced Acceptance Sampling and its Application to an Intertidal Survey 42
3.1 Introduction 42
3.2 Background to BAS 42
3.2.1 Random Numbers and Methodology of BAS 42
3.2.2 Selecting a Sample by BAS 45
3.2.3 Inclusion Probabilities and Population Estimations 46
3.3 Application of BAS to a Semi-Realistic Dataset 47
3.3.1 Population Description 47
3.3.2 Sample Selection 49
3.3.3 Spatial Coverage and Parameter Estimation 50
3.4 Further Discussions about BAS 53
3.5 Conclusions 55
3.6 References 56
Chapter 4 Population Characteristics and Performance of Balanced Acceptance Sampling 57
4.1 Introduction 57
4.2 Application of BAS on Populations With Different Spatial Autocorrelation 58
4.2.1 Using BAS in Populations Where Observations Have a Gaussian Distribution 58
4.2.2 Using BAS in Populations Where Responses are Binary Data 65
4.3 BAS for Stratified Populations 72
4.3.1 Considering Same Sampling Fraction in Each Stratum 72

4.3.2 Considering Different Sampling Fractions in Strata 77

4.4 Conclusions 81

4.5 References 82

Chapter 5 Spatially Balanced Sampling Methods for Household Surveys 83

5.1 Introduction 83

5.2 Spatially Balanced Sampling Methods in Environmental Studies Versus Household Surveys 84

5.2.1 Suitability of Balanced Acceptance Sampling for Selecting Samples From Discrete Populations 87

5.3 A Frame for BAS for Discrete Populations 89

5.3.1 Spatial Properties of the BAS-Frame Technique 95

5.3.2 Statistical Properties of the BAS-Frame Technique 98

5.3.3 Application of Spatially Balanced Sampling Methods on Real Data 104

5.4 Implementation of Spatially Balanced Sampling Methods on Stratified Populations in Household Surveys 109

5.4.1 BAS-Frame Technique for a Stratified Population 110

5.4.2 Spatially Balanced Sampling Methods When the Population is Stratified Geographically 112

5.4.3 Spatially Balanced Sampling When the Population is Stratified Demographically 115

5.5 Conclusions 123

5.6 References 125

Chapter 6 Properties of Sampling Frames for Spatial Sampling in Household Surveys 127

6.1 Introduction 127

6.2 Spatially Balanced Sampling Methods in Conducting a Two-Stage Cluster Sampling 128

6.2.1 Stage 1 – Selecting Sample Area Units in the Presence of an Area Frame 128

6.2.2 Stage 2 – Selecting Sample Households in the Presence of a List Frame 133

6.3 Spatially Balanced Sampling Methods in the Presence of a List of Household Registry 134

6.3.1 Cluster BAS-Frame Method 136

6.3.2 Application of the Cluster BAS-Frame Method 138

6.4 Application of Spatially Balanced Sampling Methods in Household Surveys in Non-ideal Situations 141

6.4.1 Selecting a Spatially Balanced Sample From a Map Using the BAS Method 142

6.5 Conclusions 147

6.6 References 148

Chapter 7 Spatially Balanced Sampling Methods and Some Features of Household Surveys 151

7.1 Introduction 151

7.2 Constructing PSUs in Household Surveys 151

7.2.1 Using the BAS-Frame Technique for Combining Undersized Neighbouring Units 154

7.2.2 Application of the Proposed Technique on the Christchurch Meshblocks 158

7.3 Spatially Balanced Sampling Methods and Longitudinal Designs 160

7.3.1 Overlap Control between Different Household Surveys 166

7.4 Spatially Balanced Sampling Methods and Availability of Auxiliary Information in the Design Stage 168

7.4.1 The Principles of LPMs and BAS-Frame in Spreading the Samples Over the Space of Auxiliary Variables 168

7.4.2 Efficiency of BAS-Frame and Number of Auxiliary Variables..... 174
7.5 Conclusions..... 183
7.6 References..... 184
Chapter 8 Conclusions and Recommendations 187
8.1 Key Contributions..... 188
8.1.1 Part 1: Practical Aspects of the BAS Method..... 188
8.1.2 Part 2: Application of Spatially Balanced Sampling in Household Surveys..... 189
8.2 Recommendations for Future Work..... 195
8.3 References..... 196
Appendix A An Algorithm for Simulating a Spatial Auto-Correlated Population 197
A.1 Introduction..... 197
A.2 Generating a Population With a Specified Spatial Auto-Correlation 197
A.2.1 Generating a Spatially Auto-Correlated Bernoulli Population 199
A.3 References..... 203

List of Figures

Figure 2-1. Two grid patterns that are usually used in two-dimensional systematic sampling: (a) a square lattice, (b) a triangular lattice. 23

Figure 2-2 the first three levels of a hierarchical quadrat partitioning in GRTS (courtesy of Stevens, D. and Olsen (2004)).....25

Figure 2-3 implementation of the Brewer and Hanif (2013) method for selecting equal and unequal probability samples.....26

Figure 2-4 An example of implementing the suboptimal LPM in a population with 15 units that have been ordered according to a relevant variable associated with the distance. Solid squares denote decided units and white squares denote undecided units. Unit $i = 7$ is selected randomly. A local subset that contains unit i's potential neighbours is selected among undecided units by considering $h = 3$. 30

Figure 2-5 The Voronoi polygons generated around sampling units in a given population with 56 units. Selected sampling units are shown enlarged. 35

Figure 3-1 The arrangement of the first 10 Halton points with $p1 = 2$ and $p2 = 3$. 44

Figure 3-2 Number of Nasima dotilliformis in the simulated population that covers 400×400 equal quadrats.....48

Figure 3-3 Moran scatter plot of number of crab burrows in the study area formed by 400 × 400 quadrats. 49

Figure 3-4 A sample of size equal to 48 quadrats drawn using (a) the BAS method, (b) the two-dimensional systematic sampling method, and (c) the simple random sampling method, respectively. 51

Figure 3-5 The $Varcomplex/VarSRS$ of two different sampling methods (BAS and SYS) with different sample sizes.....52

Figure 3-6 The resultant survey when only 30 quadrats instead of 48 quadrats are selected with (a) BAS and (b) two-dimensional systematic sampling method, respectively.....53

Figure 3-7 Examples of sampling units selected using the two-dimensional systematic sampling method from a study area with irregular shape: (a) the resultant sample size = 5, (b) the resultant sample size =3. 55

Figure 4-1 The spatial features of the generated Gaussian population with different Moran's I indices. 60

Figure 4-2 Trend of simulated variance of the HT estimator for Gaussian populations amongst different levels of Moran's I when BAS and SRS are used to select different sample sizes (a) $n = 50$, (b) $n = 100$, (c) $n = 150$, (d) $n = 200$, (e) $n = 250$, (f) $n = 300$ and (g) $n = 350$. 64

Figure 4-3 The ratio of variance of the HT estimator of the BAS method to the variance of the HT estimator of SRS, $rBAS/SRS$, for Gaussian populations amongst different levels of Moran's I. 64

Figure 4-4 Spatial features of the generated population with a Bernoulli distribution with parameter $p = 0.5$. 67

Figure 4-5 Trend of the estimated variance of the HT estimator for different levels of Moran's I in Bernoulli populations with $p = 0.5$ when BAS and SRS are used to select different sample sizes (a) $n = 50$, (b) $n = 100$, (c) $n = 150$, (d) $n = 200$, (e) $n = 250$, (f) $n = 300$ and (g) $n = 350$. 70

Figure 4-6 The ratio of the variance of the HT estimator of the BAS method to the variance of the HT estimator of the SRS, $rBAS/SRS$, for populations with Bernoulli distribution for different levels of Moran's I. 71

Figure 4-7 Study area of the population of crabs, which is partitioned into four different strata. 73

Figure 4-8 Variance of the achieved HT estimator among 1000 simulated samples selected by two different sampling designs (BAS with proportional allocation, and BAS) for a range of sample sizes. 77

Figure 4-9 Polygons (envelopes) of boundaries of samples selected by BAS with stratified sampling and StratBAS along with average values of the calculated Kr for a

range of sample sizes (a) $n = 36$, (b) $n = 81$, (c) $n = 121$, (d) $n = 169$, (e) $n = 196$, (f) $n = 256$, (g) $n = 289$. 80

Figure 5-1 Google images of (a) a discrete population (www.hnzc.co.nz) and (b) a continuous population (www.financialtribune.com). 85

Figure 5-2 A spatial sample selected from a continuous population. 86

Figure 5-3 Sampling areas selected by overlaying a grid on a small part of a city. Selected areas are shown by ✱. 86

Figure 5-4 (a) An example of a discrete population (b) equal boxes are placed around discrete units, (c) using the BAS method, a unit is selected if the Halton point is within the unit's box. The boxes of four selected sampling units are shown in red. Solid triangles show Halton points are located outside the boxes. 88

Figure 5-5 An example of a discrete population in which sampling units are located very close to each other. Very close units are shown in circles. Non-overlapping boxes around these units are so small that using BAS would be inefficient. 89

Figure 5-6 The geographical locations of 506 cases in Boston Housing Dataset. The study area is divided vertically into two boxes (B1 and B2). Since the number of units (houses) is even (506), it is not necessary to add an extra unit randomly to it. ..90

Figure 5-7 Boxes created after the horizontal division. Horizontal division is done in each box achieved in the previous step. In this example, each created box in the first step contains 253 units, so an extra unit (red points) was added randomly to each box. The current boxes are halved with the same count of units. 91

Figure 5-8 Boston Housing data study area split into 64 boxes after the first six levels of the partitioning process. During the partitioning process of the Boston Housing data into 64 boxes with the same counts of units, some units are added randomly; these units are shown in red. 92

Figure 5-9 Units in the Boston Housing dataset that have the same longitude are shown in red. Green points show units that have the same latitude. 93

Figure 5-10 A regular frame based on the primary frame shown in Figure 5-8 for selecting equal probability sampling units using the BAS method. This frame contains 64 equal-sized boxes that are addressed the same way as the primary frame. 94

Figure 5-11 (a) Selected boxes using the BAS method from a regular frame, and (b) the location of the selected boxes on the relevant primary frame.....95

Figure 5-12 An artificial population used in a spatial balance investigation of the BAS-Frame technique, overlaid with a 10 × 10 grid of square cells.....96

Figure 5-13 Comparison of spatial balance of SRS, GRTS, BAS, LPM1 and SCPS using the quadrat-based method. Results are based on using 1000 samples of size 50. The achieved sample size is the number of samples that fell into each of the 100 square cells.....96

Figure 5-14 A map of Christchurch meshblock boundaries including the centre of each meshblock. 98

Figure 5-15 Three different layers of Christchurch meshblocks showing their densities. 99

Figure 5-16 Spatial trends of the response variable y for all Christchurch meshblocks and three different layers of the Christchurch meshblocks. 101

Figure 5-17 A population consisting of 64 units which are stratified into two strata ("x" and "o").....111

Figure 5-18 The first option for sample selection from a stratified population using the BAS-Frame technique. 111

Figure 5-19 The second option for sample selection from a stratified population using the BAS-Frame technique.....112

Figure 5-20 Variance of mhi among all iterations for all evaluated designs and all sample sizes. 119

Figure 5-21 Trends of ratio of variances related to each target variable among the different sample sizes. 122

Figure 6-1 Achieved $\mu(\zeta)$ for all evaluated methods and different sampling fractions. 131

Figure 6-2 Estimated $Deffcomplex,\ CP$ for all evaluated methods and different sampling fractions. 132

Figure 6-3 Two different rules of listing paths (Redrawn from Centers for Disease Control (2010)). 133

Figure 6-4 Locations of generated housing units in two meshblocks in Christchurch. Red points show the locations of two-storey housing units. 139

Figure 6-5 An example of an area frame shown on a map. In this map, the boundaries of the area units (meshblocks) are clear. 143

Figure 6-6 Illustration of a situation that centre of an irregular shaped area is not located within the boundaries. 143

Figure 6-7 A sample of size 10 meshblocks selected by (a) BAS and (b) SRS from a map frame of a small part of Christchurch city. 145

Figure 6-8 Spatial distribution of area of some meshblocks considered in the simulation study. 146

Figure 7-1 The geographical position of units in the population described in Example 7.1. 155

Figure 7-2 (a) Vertical temporary boxes achieved after completing the first step of the division, (b) total numbers of households in each created vertical temporary box. 156

Figure 7-3 (a) Horizontal temporary boxes which are achieved after completing the second step of the division,(b) total numbers of households in each created horizontal temporary box. 156

Figure 7-4 (a) Vertical temporary boxes achieved after completing the third step of the division, (b) total numbers of households in each created vertical temporary box in the third step. 157

Figure 7-5 (a) Vertical permanent boxes achieved after completing the third step of the division, (b) total numbers of households in each created vertical permanent box in the third step. 157

Figure 7-6 (a) final boxes after completing the division process, (b) total numbers of households in each created box after completing the division process. 158

Figure 7-7 (a) Average distances (d) calculated using both methods for a range of pre-specified PSU size thresholds varying from 2 to 60 households. (b) The total distance to visit all the created PSUs. 160

Figure 7-8 The ratio of ζn for GRTS, BAS-Frame when compared to SRS and each other for a situation when sampling units are added to the sample one by one over a period of time. 162

Figure 7-9 Sample dwellings allocated into 20 different rotation groups using the BAS-Frame technique. Dwellings with same colour are in the same rotation group. 164

Figure 7-10 An example of a rotation pattern which is conducted quarterly for three successive years. Rotation groups are defined by alphabetic characters. The number of appearing of a rotation group in the sample is defined by its subscript: for example $K3$ means that rotation group K is revisited for the third time. Rotation groups that are entered to the sample for the first time are shown in grey. 164

Figure 7-11 Spatial balance of the selected sampling units in each period. 166

Figure 7-12 Sampling distribution of the auxiliary variables for three different sampling methods among 1000 samples of size 10. 172

Figure 7-13 Trend of average of spatial balance, ζ , for each sampling method amongst the number of auxiliary variables and for a range of sampling fractions: (a) sampling fraction = 7%,(b) sampling fraction = 9% and (c) sampling fraction = 10%. 177

Figure 7-14 The "scree plot" for the PCs determined from the Christchurch meshblocks dataset. 179

Figure 7-15 The simulated variances of the auxiliary variables for LPM and BAS-Frame in relation to SRS for two situations: (1) when PC1 was the only auxiliary variable in the sample selection process, and (2) when all 10 auxiliary variables were considered in the sample selection process and for three sampling fractions: (a)

sampling fraction = 7%,(b) sampling fraction = 9% and (c) sampling fraction = 10%. 182

List of Tables

Table 3-1 $\mu\zeta$, the simulated variance of the HT estimator and the estimated variance $Varest$ for two sampling schemes with different sample sizes. 52

Table 4-1 Simulated variance of the HT estimator when BAS and SRS are employed to select samples (of sizes $n = 50$, 100, 150, 200, 250, 300 and 350) from Gaussian populations with different levels of Moran's I. 61

Table 4-2 Simulated variance of the HT estimator for eight binary populations with different levels of Moran's I when $p = 0.5$ and BAS and SRS are used to select samples of size n=50, 100, 150, 200, 250, 300 and 350. 68

Table 4-3 Average and variance of the observed quadrats in each stratum for 1000 samples selected by BAS and SRS for a range of different sample sizes. Sample sizes allocated to each stratum if stratified sampling with proportional allocation were applied, are shown in rows entitled "proportional". 75

Table 4-4 Simulated variance of the achieved HT estimator for 1000 simulated samples and the average of the estimated variances for 1000 samples selected by two different sampling designs (BAS with proportional allocation and BAS). 76

Table 4-5 The average of ζ for two sampling schemes (BAS with stratified sampling and StratBAS) in different sample sizes. 78

Table 5-1 Comparison of the spatial balance of SRS, GRTS, BAS, LPM1 and SCPS using the Voronoi polygons method. The values of $\mu(\zeta)$were estimated from 1000 simulated samples and for five different sample sizes. 97

Table 5-2 Average area of meshblocks ($km2$), density, and standard deviation ($km2$) of the area of Christchurch city meshblocks ($km2$) in each layer.....100

Table 5-3 The number of meshblocks and Moran's I value of the response variables y for different layers of Christchurch city meshblocks.....100

Table 5-4 Achieved values of $\mu(\zeta)$ and $Deff$ for estimating the response variable among all Christchurch and three different layers using 1000 simulated samples with different sampling fractions using four different sampling schemes.....103

Table 5-5 Moran's I for the response variables among meshblocks in Canterbury region. 105

Table 5-6 Estimated Deff relevant to each sampling design for estimating the mean of the considered response variables. 107

Table 5-7 Average of Deff on all response variables relevant to each sampling design for estimating the average value of the considered response variables. 108

Table 5-8 Average of ζ among all 1000 replicates for the evaluated sampling design. 109

Table 5-9 Calculated d(km) among 1000 repetitions for the evaluated sampling methods in urban and rural strata. 114

Table 5-10 Calculated relative distance corresponding to LPM and BAS-Frame technique, for urban and rural strata, and for three different sampling fractions. 114

Table 5-11 The average and variance of mhi among all iterations for all evaluated designs and all sample sizes. 118

Table 5-12 The ratio of variance related to each target variables and for all considered sample sizes. 121

Table 6-1 Achieved $\mu(\zeta)$ and $Deffcomplex,\ CP$using 1000 samples for different sampling fractions. 131

Table 6-2 Simulated variance of HT estimator for estimating households' average income and the shortest distance (km) for visiting the selected sample among 1000 samples selected by the Cluster BAS-Frame and BAS-Frame method for a range of sampling fraction. 140

Table 6-3 The average of ζ among 1000 samples selected by four different sampling methods (i.e., BAS on a map frame, SRS on a map frame, BAS-Frame using centre of areas, and SRS) and for five different sampling fractions. 147

Table 7-1 Geographical units along with their sizes. 153

Table 7-2 Geographical units which either itself or its next following unit has less than 25 households in the considered population. 153

Table 7-3 The average of ζ among 1000 iterations for BAS-Frame and LPM in comparison with the relevant value for SRS. 171

Table 7-4 The simulated variance of the total estimation of the variables of interest where samples are selected by LPM1 and BAS in relation to SRS.....173

Table 7-5 List of auxiliary variables in each stage of the simulation study. 175

Table 7-6 The ratio of the average of ζ for the spatially balanced sampling methods when compared to the relevant values achieved from SRS, for each sampling fraction and number of considered auxiliary variables. 176

Table 7-7 The ratio of the average of ζ for the spatially balanced sampling methods when compared to the relevant values achieved from SRS.....180

Table 7-8 The simulated variances of the auxiliary variables for LPM and BAS-Frame in relation to SRS for three sampling fractions (7%, 9% and 10%) and two situations: (1) when PC1 was the only auxiliary variable in the sample selection process, and (2) when all 10 auxiliary variables were considered in the sample selection process. 181

Chapter 1 *Introduction*

1.1 Background

Demand for reliable and detailed information about demographic and socio-economic characteristics of households and individuals has increased dramatically over the past few decades. In general, demographic and socio-economic information is collected from each individual in a population by conducting a population and dwelling census, hereinafter referred to as a census. Although censuses are major sources of baseline data on demographic and socio-economic characteristics, they may not be able to respond to all the varied demands for information. Censuses are usually carried out at five- or ten-year intervals, and information from intermediate years may be required. In addition, it is not usually feasible to use censuses to cover a range of different subject matter in detail. Because of these reasons, and in an attempt to reduce the cost of data collection, the method of generating demographic and socio-economic information has transformed from the full census enumeration to the theory of sampling surveys (Kruskal & Mosteller, 1979).

In many countries, household sampling surveys are the most common mechanism for obtaining the required information in socio-economic studies. Undoubtedly, increasing the efficiency of household sampling surveys can lead to more reliable estimation of socio-economic factors in a population. It is critical, therefore, for statisticians and statistical agencies to explore new ways that make household sampling surveys more efficient.

Consideration of the spatial properties of sampling units has led to a new area of sampling methodology entitled spatially balanced sampling (Wang, J.-F. et al., 2012; Benedetti et al., 2017). The idea behind spatially balanced sampling is to spread the sampling units evenly over a region corresponding to the population of interest. Although spatially balanced sampling has been developed over the past few years, its application is relatively new in socio-economic surveys.

This PhD research intends to assess the possibilities for the application of spatially balanced sampling in socio-economic studies and, in particular, household surveys. Household surveys are important as they are widely used for collecting a range of social and economic information (e.g., income, employment, education, health, political opinion) from the population.

1.2 Research Motivations

The current methodologies for conducting household surveys use standard and rather elementary sampling designs. Recent studies (Brown et al., 2015; Grafström & Schelin, 2014) have shown that spatially balanced sampling methods—which provide a good spatial coverage of the population of interest— can be employed for selecting representative samples. This research is intended to examine the applicability of these methods in household surveys in an attempt to enrich the range of sampling designs.

Household surveys are typically multi-objective surveys that provide estimates for a vast variety of variables of interest (e.g., unemployment rate, median income, etc.). They usually use stratified sampling methods to achieve more precise estimates and include various sub-groups of interest in the sample. However, choosing appropriate variables for constructing homogenous strata and allocating sample size to each stratum are some of the most challenging issues raised in conducting a multi-objective household survey. Because spatially balanced sampling methods can use the geographical coordinates associated with population units (Benedetti et al., 2017), the motivation for this research was to investigate whether these methods can be used instead of a conventional stratified sampling method in multi-objective surveys.

A complete and reliable list of households (or individuals) is usually unavailable, and hence household surveys typically employ a multi-stage sampling design in which sampling is done sequentially through two or more hierarchical stages (Chauvet, 2015). Sampling units at the early stages of a multi-stage sampling design in household surveys are drawn from a list of geographical areas (e.g., counties, postcode areas or blocks), while the sampling units at the last stage need to be selected from a list of dwelling units in the selected geographical areas.

To date, the common practice of preparing a frame for the last stage of a multi-stage sampling design in a household survey has been to send interviewers to the

sampled geographical areas in order to create a list of dwelling units. The development of these frames may be expensive and time-consuming, therefore national statistical agencies have been motivated to replace these frames with a new form of sampling frame, namely a list of residential postal addresses (Kalton et al., 2014; Valliant et al., 2014; Australian Bureau of Statistics, 2014).

Since address-based frames (based on a list of residential postal addresses) will support household surveys in the future, this study proposes a technique to use spatially balanced sampling methods for selecting samples from a list of registered housing units. Furthermore, in some cases, household sampling surveys need to be conducted in non-ideal situations (e.g., when sampling frames are not available). This may be the case when conducting household surveys in poorly resourced countries or after a disaster. This is another motivation for this thesis: to investigate how spatially balanced sampling can be modified to be used in these non-ideal situations.

Over the last few decades, there has been a growing tendency to conduct household surveys to monitor population characteristics over time. This highlights the importance of using longitudinal designs, because they allow for collecting data over periods of time. Longitudinal designs involve a sequence of samples, which may or may not overlap in time (Elliot et al., 2009). One important type of longitudinal design is rotation panel sampling, which is extensively used in household surveys (Steel, 1997). In rotation panel sampling, a portion of sampling units is replaced with new sampling units on each fieldwork occasion (e.g., months, quarters and years). Groups of sampling units that are visited on the same fieldwork occasion are called rotation groups and should have no overlap with each other. Overlap between rotation groups is conventionally controlled by providing a master sampling frame that is updated regularly and allocating its units systematically to the rotation groups (Steel, 1997).

Spatially balanced sampling methods have the potential to select samples with no overlap and this aspect motivated this research to examine its applicability in conducting a rotation panel sampling in order to control overlap between rotation groups.

There have been few studies on the application of spatially balanced sampling methods to household surveys (Kumar, 2007; Kondo et al., 2014). This thesis will

contribute to understanding their use in household surveys. The focus is on the implementation of a spatially balanced sampling method called Balanced Acceptance Sampling (BAS; Robertson et al., 2013) as this sampling method is based on a relatively simple algorithm, and can easily be used in a large population.

1.3 Research Objectives and Scope of Work

The objectives of this thesis are as follows:

Part 1. To explore the characteristics of BAS from a practical standpoint by:

- Applying the BAS method to selecting samples from a semi-realistic dataset and identifying the potential benefits of implementing BAS in practical cases.
- Investigating the effect of the spatial autocorrelation of the population units on the efficiency of BAS (i.e., in terms of precision of estimates of the parameters of interest).
- Examining the applicability of the BAS method as a tool for selecting a sample from stratified populations.

Part 2. To promote the application of spatially balanced sampling in household surveys by:

- Exploring the dissimilarities of the implementation of spatially balanced sampling in environmental studies compared with household surveys.
- Introducing a technique that increase the efficiency of using the BAS method in finite (discrete) populations where the units are located over the space irregularly by:
    - Comparing the efficiency of the proposed technique, in terms of providing more precise estimates, with available spatially balanced sampling methods in the literature.
    - Studying the application of the proposed technique on a real dataset.
    - Investigating the application of the proposed technique for selecting samples from stratified populations.

- Investigating the effect of the sample-frame properties in household surveys on the applicability of spatially balanced sampling methods by:
    - Assessing the applicability of spatially balanced sampling methods in selecting samples from frames that are conventionally used in multi-stage sampling designs for household surveys (i.e., a list of geographical areas known as area frames at the early stages, and a list of dwelling units known as list frames at the last stage).
    - Introducing a technique to use the BAS method for selecting samples from address-based frames (i.e., a list of residential postal addresses).
    - Investigating the application of the BAS method when the only available frame is a map-based frame (i.e., a map of geographical areas).
- Studying the applicability of the BAS method to design primary sampling units (PSUs) in a multi-stage sampling design for household surveys.
- Investigating the applicability of the BAS method in longitudinal designs in household surveys.
- Assessing if the incorporation of information from auxiliary variables would limit the use of spatially balanced sampling methods in household surveys.

1.4 Organization of Thesis

The thesis consists of eight chapters, including Introduction, Conclusions and six core chapters.

Chapter 2 summarises the main literature relevant to this thesis. An introduction to the theory of sampling design and the important features of household sampling surveys are discussed. The concept of spatial autocorrelation in a spatial population and the methodology of some spatially balanced sampling methods are reviewed. Finally, the indices used for comparing the efficiency of spatially balanced sampling in terms of providing spatially balanced samples are summarized.

Chapter 3 discusses the implementation of the BAS method for practical settings in environmental studies, where the aim is to gather information from a continuous or regular discrete population. A comprehensive review of the theory of the BAS method and its application to a case study of crustaceans is presented. The dataset used in this chapter contains information on crabs from Alkhor, Qatar. On completing the study, the results associated with the implementation of BAS are compared with a two-dimensional systematic sampling method — a common sampling method in environmental studies.

Chapter 4 presents the applicability of the BAS method in the presence of populations with different characteristics. The first part discusses the effect of the distribution of a unit's response variables on the efficiency of the BAS method in terms of the precision of estimates of the parameters of interest. To this end, some artificial data sets were generated, with different levels of spatial autocorrelation from two types of populations: a population where the units follow a Gaussian distribution, and a population with binary responses. In the second part, application of the BAS method on stratified populations is assessed. A simulation study is conducted to understand whether the BAS method can be used as an alternative to the stratified sampling method when samples are selected from strata with an equal sampling fractions.

Chapter 5 provides a more detailed investigation on the applicability of spatially balanced sampling in household surveys. This chapter introduces a technique (called BAS-Frame) for implementing the BAS method in a discrete population including where the units are located irregularly over the region of interest. This chapter also presents the application of the BAS-Frame and other spatially balanced sampling methods for selecting samples from a list of meshblocks (i.e., the smallest geographical area defined by Stats NZ) in New Zealand. The implementation of spatially balanced sampling methods for selecting samples from rural and urban areas in New Zealand is also investigated. Finally, this chapter discusses the potential benefits of implementing spatially balanced sampling methods in multi-objective household surveys that aim to optimize the sample design for a list of variables of interest.

Chapter 6 discusses the role of sampling frames for applying spatially balanced sampling methods in household surveys. Applicability of the spatially balanced

sampling methods is investigated in three different situations: (a) an ideal situation when there is an area frame, (b) an ideal situation when there is a list frame, and (c) a non-ideal situation where a reliable frame is not available. For the first two situations, the spatially balanced sampling methods are compared with the conventional sampling techniques (simple random sampling, systematic sampling, proportional to size sampling) in a two-stage cluster sampling. For the third situation, the implementation of the BAS method on a map-based frame is studied. A new technique is presented, which can be used for selecting samples from a list of housing postal addresses. The efficiency of this technique is investigated through conducting a simulation study on an artificial population generated from information on the meshblocks.

Chapter 7 investigates the applicability of spatially balanced sampling to deal with some practical aspects of designing a household survey. In the first part, a procedure on the basis of the BAS-Frame method for combining undersized PSUs is presented. The implementation of spatially balanced sampling in longitudinal designs in household surveys is also discussed. Finally the incorporation of information from auxiliary variables on the efficiency of spatially balanced sampling methods is assessed.

Chapter 8 presents the overall conclusions to the research and discusses possible extensions and future work.

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Chapter 2 *Sampling Design Approaches*

2.1 Introduction

In sampling, a subset of the target population, the "sample", is selected according to specific rules, the "sampling design". After collecting information from this sample, the results are generalized to make inferences about the whole population (Hájek, 1959). Different objectives of a survey and the properties of the population under study necessitate the application of different sampling designs (Cochran, 1977; Levy, P. & Lemeshow, 2013; Särndal et al., 2003). For instance, a proper sampling design to study the labour force structure in a city may be completely different from the sampling design suitable for a study investigating the prevalence of respiratory disease in a city.

Selecting appropriate sampling designs for socio-economic studies is very important for both the statistical precision of the estimates and for practical and financial aspects. In many countries, household surveys are the most common mechanism to obtain the required information in socio-economic studies. Undoubtedly, increasing the efficiency of household sampling surveys can lead to more reliable estimation of key socio-economic factors of the population; therefore, it is critical for statisticians and statistical agencies to explore new ways that make household sampling surveys more efficient.

As already discussed in Chapter 1, the purpose of this thesis is to investigate the suitability of spatially balanced sampling methods in socio-economic studies focused on household surveys. This chapter provides a review of the theoretical framework and background that will be used or further developed in the following chapters. After an introduction to sampling design theory and common sampling methods, the properties of household sampling surveys as the main tool for generating socio-economic information will be discussed. Then, some recently developed sampling methods will be introduced that consider the geographical locations of population units in the sample selection process.

2.2 Essential Concepts and Notations

A survey population in socio-economic studies is often defined by a set of N identifiable units which may be labeled with numbers $i = 1, 2, \dots, N$.

$$U = \{1, 2, \dots, N\}$$

With each unit i in the population, there is an associated value Y_i. The values Y_i can be numerical, categorical or ordinal values. Since the statistical inferences that will be used in this thesis rely on design-based techniques, the values of Y_i are considered to be fixed, but unknown quantities.

Often a specific function of the population values, say a parameter $\theta(Y_1, \dots, Y_N) = \theta(\boldsymbol{Y})$, is unknown. Sampling surveys aim to obtain unbiased and precise estimates of parameters through suitable sampling designs (Cochran, 1977; Thompson, 1997; Lohr, 2009). The common parameters in socio-economic studies include the population total given by

$$T = \sum_{i \in U} Y_i \tag{2.1}$$

and the population mean given by

$$\bar{Y} = N^{-1} \sum_{i \in U} Y_i \tag{2.2}$$

Once a subset of the population, s, is selected, the observed data can be used to estimate the unknown parameters in the population. Assuming $y_1, \dots, y_n$ are sampling survey data of size n on the variable y, let $\hat{\theta} \coloneqq \hat{\theta}(y_1, \dots, y_N)$ be an estimator of the parameter of interest θ.

As mentioned above, a sampling survey is characterized by a combination of a sampling design and an estimator of the parameter of interest (Hájek, 1959). In this thesis, more attention will be paid to the choice of sampling design while attention to the estimator methods is restricted to the use of some simple estimators such as the Horvitz–Thompson (HT) estimator of total (Horvitz & Thompson, 1952).

2.3 Probability Sampling Design

With regard to the techniques for selecting the sampling units, the sampling designs can be classified as probability or non-probability sampling. In probability sampling designs, sampling units are selected from the population based on randomization. In fact, in probability sampling designs each unit in the population is assigned a known non-zero probability of being including in the sample, which enables statisticians to make inferences about the population based on the selected sample (Cochran, 1977; Tillé, 2006). Randomness in selection of sampling units in socio-economic surveys can increase the chance of obtaining a more representative sample (United Nations-Statistical Division, 2008). In contrast, in non-probability sampling designs, samples are not selected based on randomization. In these cases, sampling units are usually selected on the basis of their accessibility or personal judgment of the researcher (Cochran, 1977).

This thesis focuses on probability sampling designs, which are suitable for socio-economic surveys. In a finite population of size N, with a probability sampling design, unit i is assigned an inclusion probability π_i such that

$$0 \leq \pi_i \leq 1 \quad \& \quad \sum_{i=1}^{N} \pi_i = n \tag{2.3}$$

where n is the sample size.

Given inclusion probabilities for all units in the population, an unbiased estimator for $T = \sum_{i=1}^{N} Y_i$ is the Horvitz-Thompson (HT) estimator, given by

$$\hat{T}_{HT} = \sum_{i=1}^{N} \frac{Y_i}{\pi_i} I_i \tag{2.4}$$

where I_i is 1, if the i^{th} unit in the population is selected in the sample, and 0 otherwise.

The variance of $\hat{T}_{HT}$ is

$$V(\hat{T}_{HT}) = -\frac{1}{2} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ (j \neq i)}}^{N} (\pi_{ij} - \pi_i \pi_j) \left(\frac{Y_i}{\pi_i} - \frac{Y_j}{\pi_j} \right)^2 \tag{2.5}$$

where π_{ij} is the second order inclusion probability, i.e., the probability of including both units i and unit j in a sample of size n.

Using the unbiased Sen–Yates–Grundy estimator, the variance of $\hat{T}_{HT}$ in Equation (2.5) can be estimated as below

$$\hat{V}_{SYG}(\hat{T}_{HT}) = -\frac{1}{2}\sum_{i=1}^{N}\sum_{\substack{j=1 \\ (j\neq i)}}^{N}\frac{(\pi_{ij}-\pi_i\pi_j)}{\pi_{ij}}\left(\frac{Y_i}{\pi_i}-\frac{Y_j}{\pi_j}\right)^2 I_{ij} \tag{2.6}$$

where I_{ij} is 1 if both the i^{th} and j^{th} units are selected in sample and 0 otherwise.

When all population units are assigned an equal inclusion probability (i.e., $\pi_i = n/N$ for $i = 1,2,\dots,N$), all units have an equal chance of being selected in the sample. This is called equal probability sampling or an equal probability selection method (EPSEM) (Hansen et al., 1953). In practical settings, especially in socio-economic studies where sampling units vary in their importance, an equal probability sampling might result in an unfortunate selection of only less important units, with none of the highly important units included. Hence, the estimates from such a sample may be misleading.

Unequal probability sampling is one solution to achieve a more informative sample. In unequal probability sampling, units in the population are assigned different inclusion probabilities (e.g., based on their expected response values where the units with higher expected response values have a higher chance of being selected in the sample). Unequal probability sampling designs can result in more precise estimates by assigning higher inclusion probability to more important units (Thompson, 1997).

2.4 Review on Some Classic Sampling Designs

Simple random sampling (SRS), stratified sampling and cluster sampling are three major probabilistic sampling designs that play important roles in socio-economic surveys. Since these methods form the basis for extending new sampling methods, they are reviewed briefly in this section. Skinner et al. (1989), Thompson (1997), Särndal et al. (2003), and Chambers and Skinner (2003) provide full theoretical details about these sampling designs. Their practical aspects can also be found in Lehtonen and Pahkinen (2004), and Korn and Graubard (2011).

2.4.1 Simple Random Sampling

SRS is the simplest probability sampling method: it can choose a random sample with or without replacement of size n. In this thesis we use SRS without replacement (SRSWOR). With SRSWOR, each unit is assigned an equal chance (probability) of being included in the sample, $\pi_i = n/N$ for $i = 1, 2, \dots, N$. So the HT estimator and its variance can respectively be rewritten as

$$\widehat{\mathrm{T}}_{\mathrm{HT-SRS}} = \frac{\mathrm{N}}{\mathrm{n}} \sum_{i \in s} y_i \tag{2.7}$$

$$\mathrm{V}\left(\widehat{\mathrm{T}}_{\mathrm{HT-SRS}}\right) = \mathrm{N}(\mathrm{N} - \mathrm{n}) \frac{S_y^2}{\mathrm{n}} \tag{2.8}$$

where S_y^2 is the variance of the population, which is given by

$$S_y^2 = \frac{1}{N-1} \sum_{i \in U} (Y_i - \bar{Y})^2 \tag{2.9}$$

An unbiased estimator of Equation (2.8) is given by

$$\hat{V}\left(\hat{T}_{HT-SRS}\right) = N(N-n) \frac{s_y^2}{n} \tag{2.10}$$

where s_y^2 is the variance of the selected sample, which is given by

$$s_y^2 = \frac{1}{n-1} \sum_{i \in s} (y_i - \bar{y})^2 \tag{2.11}$$

where $\bar{y} = \frac{1}{n} \sum_{i \in s} y_i$ is the sample mean of the variable of interest.

Although SRS is a straightforward sampling design in theory, it may be difficult to perform in practice due to the need for a complete list of eligible population units. SRS is a baseline for developing other complex sampling designs. Also, it is often used as a benchmark for comparing the relative efficiency of other sampling techniques (Lohr, 2009; Levy, P. & Lemeshow, 2013). This comparison can be done by calculating the ratio of the variance of the parameter of interest in the complex sampling design to that in SRS of the same size. This ratio, which is referred to as "design effect" ($Deff$) expresses how much larger the sampling variance for the complex survey is compared with a simple random sample of the same size:

$$Deff = \frac{V(\hat{\theta}_{Complex\ Survey})}{V(\hat{\theta}_{SRS})} \tag{2.12}$$

If a complex survey has a higher variance compared with SRS then $Deff > 1$ and the complex survey would be considered to have lower precision. In contrast, if $Deff < 1$, it shows that the complex survey would have a smaller variance compared with SRS and therefore would be more precise.

2.4.2 Stratified Sampling

In socio-economic surveys, there is often additional auxiliary information about the population units (individuals or households) that can be used to partition the population into homogeneous subgroups (strata). Examples of subgroups are ones based on geographical boundaries such as rural versus urban, or non-geographical measures such as age, gender, or employment status.

Stratified sampling is a classic sampling technique that uses auxiliary information to increase the efficiency of a sampling design by selecting a more representative sample across all the identified subgroups (Tschuprow, 1923; Neyman, 1934). In stratified sampling, once the strata have been defined, sampling units within each stratum are selected independently of the other strata (Groves et al., 2011). The overall sample estimates are calculated from the weighted sum of the stratum estimates. The sampling fraction in each stratum, that is, the ratio of the sample size to the size of the stratum (Dodge & Marriott, 2003) can be controlled by allocating sample units to each stratum. "Equal allocation", "proportional allocation", "square root allocation" and "Kish allocation" are common allocation methods for determining the strata sample size, see Cochran (1977), Kish (2004) and Lohr (2009) for more details of explicit stratification and sample size allocation to each stratum.

Selecting proper stratification variables that are strongly correlated with the variable of interest increases both the homogeneity within strata and heterogeneity between strata. This reduces the sampling error, and consequently increases the precision of the estimates (Cochran, 1977). However, if strata are chosen with no regard for the variable of interest, it is possible that the variance of the parameter estimator is not reduced by stratified sampling, compared with SRS. This may happen in large multipurpose surveys that aim to meet several objectives. In these situations, there is a

possibility that the preferred stratification variable for a certain objective would not be relevant to other objectives.

This problem may also arise in longitudinal surveys in which sampling units are followed over a considerable period of time and where the strata boundaries are changed. In fact, there is no guarantee that the demographic variables that are commonly used as stratification variables remain constant over time. Hence, the boundaries of strata might be unsuitable in the latter periods of observation in a longitudinal survey.

This thesis will investigate the suitability of using the geographical location of population units as an auxiliary variable in conducting multipurpose and longitudinal surveys.

2.4.3 Cluster and Multistage Sampling

Cluster sampling is a procedure to select sampling units from a population whose units are naturally grouped together into clusters. Cluster sampling is done in a two-step process whereby, in the first step, a sample of clusters is selected randomly and then in the second step, all or a subset of units within the selected clusters are visited as sampling units (Cochran, 1977). Cluster sampling is usually less precise than SRS, but it is performed in most of the large-scale socio-economic surveys such as household surveys (Harter et al., 2010).

Reducing the survey cost is the first and main reason for using cluster sampling in socio-economic surveys that are based on personal contact. For example, it is usually more cost-effective to observe 500 individuals in 10 clusters (50 units per cluster) than to visit 500 individuals selected randomly throughout the population.

In addition, extracting a sample of individuals (for example, households) directly from a population needs a complete and suitable list of all eligible individuals. One way to construct such a list is to enumerate all individuals in the population, an expensive and time-consuming task. In this situation, cluster sampling may reduce the time and costs by selecting some geographical regions as sample clusters, and then creating a list of individuals in the sampled clusters – who were selected in the first stage – instead of enumerating the entire population.

Cluster sampling can be extended into a more complicated format that selects sampling units in more than two stages, hierarchically. This sampling scheme is called multistage sampling. In the first stage of the multistage sampling method, some units, termed "primary sampling units" (PSUs) in sampling literature, are selected randomly. In the second stage, some units are sampled from the selected PSUs. These units are called second-stage units or "secondary sampling units" (SSUs); units selected from the sampled SSUs at the third stage are referred to as the third-stage units, and so on. In the end, the units that are selected at the last stage are known as "ultimate sampling units" (USUs).

Decisions on which units can be used as PSUs and the way they should be selected are important aspects in using multistage sampling in socio-economic surveys. A special form of multistage sampling that is commonly used in socio-economic surveys is area sampling. In area sampling, geographical areas such as counties, townships, and city blocks, are visited as the intermediate units to access the target units (households or individuals) in lower levels (Valliant et al., 2013).

2.5 Sampling Designs in Household Surveys

Household surveys are the most common type of sampling surveys that are carried out by statistical agencies to obtain social and demographic information about the population of interest. Sampling designs for household surveys in most countries have many similar features. Generally, they are complicated designs as they usually include multistage sampling, stratification and unequal selection probabilities in each stage (Yansaneh, 2005). Furthermore, most household surveys are multipurpose in scope, and this increases their complexity. Commonly used sampling methods in household surveys are stratified multistage probability sampling designs. These sampling strategies rely on the advantages of both stratification and multistage sampling to increase the survey efficiency and decrease the survey cost (Som, 1973).

In stratified multistage sampling, the population is partitioned into strata according to relevant available auxiliary variables. Then, the sample selection process is hierarchically carried out within the strata. In each stratum, the numbers of stages, and number of sampling units in each stage, are varied in different surveys according to the

target population, objectives of the survey, and prevalence rates for specified population characteristics.

The idea of selecting sampling units (households or individuals) in socio-economic studies using multistage methods has been around for many years (Murphy, 2008). In 1802, in order to estimate the population of France, Laplace suggested to sample a subset of small administrative districts known as communes instead of enumerating the entire population. After counting the total population in communes (y), the population of France was estimated by $(B \times \frac{y}{b})$, where b and B are the known number of births in communes and nation, respectively (Wright, 2001). In 1895, A.N. Hiaer, the Norwegian statistician, proposed a method for socio-economic surveys by selecting a portion of towns or districts in the first step and then selecting sub-units systematically within the selected towns or districts in the second stage (Wright, 2001). National Health Interview Survey (NHIS), Household Expenditure and Income Survey (HEIS), and Labor Force Survey (LFS) which are conducted in most countries rely on stratified multistage sampling.

According to the suggestions of the United Nations for designing household surveys (Pettersson, 2005), the area units are generally formed into PSUs. After constructing the PSUs, some of them are selected randomly in the first stage with probability proportional to some measure of size variables, such as estimated or known number of households or individuals. The process of selection continues for two or three stages and finally, ultimate units (households or individuals) are selected randomly within each sampling unit selected in the recent stage.

Sample selection in the first-stage of a stratified multistage sampling is performed in three major steps including defining PSUs, stratifying PSUs, and sampling the PSUs. These steps are explained below.

2.5.1 Defining PSUs in Household Surveys

As mentioned before, PSUs in household surveys are often composed of geographic areas. For instance, PSUs in the Household Labour Force Survey (HLFS) in New Zealand are aggregations of one or more meshblocks, which are the smallest units of geographical area in New Zealand (Stats NZ, 2017). In the Current Population Survey

(CPS) in the United States, each PSU consists of either a single county or two or more contiguous counties (United States Bureau of the Census, 2000).

PSUs play an important role in the quality of the sampling survey process, so particular attention should be dedicated to defining them carefully. Yansaneh (2005) introduces some properties of a suitable PSU, as listed below:

a) Its boundaries should be identified clearly to ensure stability of the PSU over time.
b) All PSUs together have to cover the population completely.
c) The PSU should have a measure-of-size variable for conducting unequal probability sampling methods during the sampling process.
d) The PSU should have some auxiliary variables for stratification purposes.
e) The PSU should be large enough to prevent exhaustion problems when the PSU has to be used extensively. The size of a PSU depends on the predetermined workload in the survey. For example, each PSU in CPS has a population of at least 7500 (United States Bureau of the Census, 2000), and Statistics New Zealand considered an average of 70 occupied and under-construction dwellings as the size of PSUs constructed from the 2013 Census (Stats NZ, 2017).

In order to form desirable PSUs, usually very small natural geographical areas (such as meshblocks or counties) are combined with their neighbours and very large ones are divided into a number of reasonably sized subregions called segments.

Combining small PSUs is harder than partitioning large ones. Kish (1965) introduced a procedure for combining PSUs, but this procedure does not guarantee that the selected PSUs for grouping are contiguous. This thesis addresses this drawback by using a partitioning technique that is used in some spatially balanced sampling methods. The proposed technique for defining PSUs with desirable sizes will be discussed in Chapter 7.

2.5.2 Stratifying PSUs in Household Surveys

Once PSUs have been defined, they are stratified using a set of geographical and socio-economic variables. Selecting a set of these auxiliary variables correlated with the key variable of interest is an important part of experimental design.

There are some software packages that can be used to perform the PSU stratification in household surveys. The CPS, for instance, uses the stratification search program (SSP), created by Bureau of Labour Statistics in United States, to perform the PSU stratification (Thurgood et al., 2003; Murphy, 2008).

Stratification techniques are further discussed in Chapter 5 in the context of applying spatially balanced sampling methods to household surveys.

2.5.3 Sampling PSUs in Household Surveys

After stratifying PSUs, the probability sampling method can be used to select PSUs randomly from each stratum with probabilities usually proportional to some measure of size such as total population or the number of households within PSUs. This method is called probability proportionate to size (PPS) sampling.

In order to create a reasonable geographical spread, the PSUs are usually listed in some kind of geographical order (e.g., cities in a province) and then systematic selection is applied. In Chapter 6, spatially balanced sampling methods are used as alternative sampling methods for selecting PSUs, and the efficiencies of these methods are compared with the sampling methods that are currently used for selecting sample PSUs.

2.5.4 Longitudinal Designs in Household Surveys

Longitudinal or repeated designs are a type of survey that collects data from the same sampling units over a period of time (monthly, quarterly or yearly) to measure changes of some population characteristics (Binder, 1998). Monitoring socio-economic indicators and detecting changes in time is the main reason for utilizing repeated designs (Steel & McLaren, 2009). A special case of repeated surveys used in most statistical organizations is a rotating panel that replaces a predetermined proportion of the sample with new units on each occasion. Some examples of rotating panel designs

include quarterly labour force surveys in the United States, Australia, New Zealand, Iran, Canada and Japan.

Defining the rotation pattern and the rotation groups are two major aspects in designing a rotating panel survey. Both of these aspects are dictated by the nature of the survey characteristics and the number of times the agency can revisit a unit (Smith, P. et al., 2009). Two frequent rotation patterns are "in-for-T" and "T-O-T". In the first, a sample unit is revisited for T successive occasions and then left out from the sample. In the second, a unit is in the sample for T successive occasions, left out for O successive occasions and then it is sampled again for a further T successive occasions (Steel, 1997). For example, the CPS in the United States and HLFS in New Zealand have "4-8-4" and "in-for-8" designs, respectively.

In order to manage rotation of the sample between different occasions, the sampling units are split into rotation groups. In order to avoid the selection of adjacent housing units in the same rotation group, the housing units can be systematically allocated in different rotation groups. For example, in a PSU consisting of 50 dwellings $(d_1, d_2, \dots, d_{50})$, the 5 rotation groups can be defined by

Rotation group 1: $d_1, d_6, \dots, d_{46}$
Rotation group 2: $d_2, d_7, \dots, d_{47}$
Rotation group 3: $d_3, d_8, \dots, d_{48}$
Rotation group 4: $d_4, d_9, \dots, d_{49}$
Rotation group 5: $d_5, d_{10}, \dots, d_{50}$

In Chapter 7 of this thesis I will explain how the spatially balanced sampling method can produce different independent rotation groups in longitudinal designs.

2.6 Spatial Autocorrelation and *Moran's I* Index

The importance of considering the geographical relationship between sampling units was first pointed out by Francis Galton in his exchange with Tylor (1889) over Tylor's presentation, in which Tylor had gathered information on institutions of marriage and descent for 350 cultures. After analysing the data, he interpreted his results as indications of a general evolutionary sequence. Galton's criticism was that the observations that were collected across areal entities were not independent observations. Galton's critique has since been known as Galton's Problem (Naroll,

1961, 1965; Naroll & D'andrade, 1963). Later, Cruickshank (1940, 1947) considered the principle of nearness in a study of the incidence of human cancer in different parts of England and Wales. Cox's work (1969) in investigating the influence of areal contiguity on the percentage of Democrat votes is another example that emphasises the relationship between nearby areas.

When adjacent units are more likely to have characteristics similar to their neighbours, it is said that the adjacent units exhibit spatial autocorrelation (Fortin et al., 2002). In this situation, response values corresponding to units at a specific location in space are not independent of response values corresponding to units at locations nearby (Dow et al., 1984). In other words, spatial autocorrelation expresses the dependency between values of a variable in close proximity to each other (Griffith, 2009). For example, households that are near each other tend to have similar incomes, and the health status of households in a given air pollutant area is similar.

Spatial autocorrelation may be measured in various ways. One of the most well-known indices used to quantify the spatial autocorrelation was introduced by Moran (1948).

Moran's I is structured similarly to the Pearson's correlation coefficient[1], but by substituting the neighbouring value of the variable of interest instead of the second variable:

$$I = \frac{N}{\sum_i \sum_j w_{ij}} \frac{\sum_i \sum_j w_{ij}(Y_i - \bar{Y})(Y_j - \bar{Y})}{\sum_i (Y_i - \bar{Y})^2} \, , i \neq j \tag{2.13}$$

where N is the number of observed spatial units; Y_i and Y_j are the values of the variable of interest (Y) related to units i and j respectively; and $\bar{Y}$ is the average of Y over the entire population. The term w_{ij} is the element of the matrix of spatial weights $\boldsymbol{W}$specifying the strength of the relationship between any two spatial units i and j. The diagonal elements of $\boldsymbol{W}$ are all equal to zero (i.e., $w_{ij} = 0$) .

[1] Pearson's correlation coefficient between two variables x and y is measured by $r = \frac{\sum_{i=1}^{N}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{N}(X_i - \bar{X})^2}\sqrt{\sum_{i=1}^{N}(Y_i - \bar{Y})^2}}$, where N is the number of observations.

There are a number of ways to define the weights matrix $\boldsymbol{W}$ via similarity measures. Generally, it is defined using contiguity between units. The two most common forms of contiguity in spatial studies are "rook" and "queen" definitions of neighbours. In the rook relationship, the units that share a common boundary with the unit of interest are considered neighbours of that unit, whereas a queen relationship means that the units adjacent via either side or corner are the unit's neighbours. As another option, the weights matrix W may represents the inverse distance between units.

The value of *Moran's I* usually lies between -1 and $+1$, but sometimes falls outside that range (Legendre & Fortin, 1989). Under the assumption that the data are independent and identically distributed normal random variates, *Moran's I* is asymptotically normally distributed and, under the null hypothesis of no spatial autocorrelation, its expectation is $(I) = -\frac{1}{N-1}$, which tends to zero as N increases. Positive spatial autocorrelation is indicated by values greater than $E(I)$, whereas values less than $E(I)$ imply negative spatial autocorrelation (Griffith, 1987).

In Chapter 4 of this thesis I will use the *Moran's I* index in order to characterise synthetic populations with different levels of spatial autocorrelation. *Moran's I* will also be employed in Chapter 5 to describe the spatial autocorrelation among units of the population under study.

2.6.1 Review of Some Spatially Balanced Sampling Methods

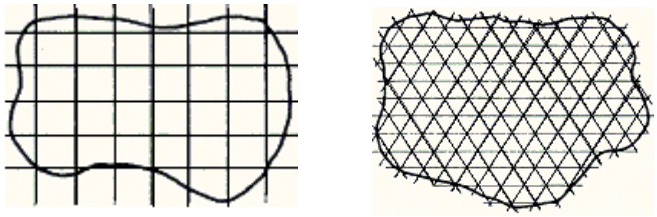
In the presence of spatial autocorrelation, neighbouring units tend to have similar values of the variable of interest and therefore provide little additional information.

Spatially balanced sampling methods are sampling designs that avoid selecting neighbouring units in the same sample. These sampling methods, which are popular in environmental studies, can improve the efficiency of population estimates by selecting a sample with few nearby units (Theobald et al., 2007). Here, some well-known spatially balanced sampling methods which are used in the next chapters are reviewed.

2.6.1.1 Two-Dimensional Systematic Sampling

Two-dimensional systematic sampling selects the initial sampling unit randomly and uses it as the origin for a regular pattern, over which the rest of the sampling units are located. Payandeh (1970) and Tomppo and Heikkinen (1999) have used two-dimensional systematic sampling in forest surveys estimating the abundance of trees in a given region. Many other examples of two-dimensional systematic sampling can be found in soil sampling (Mason, 1992).

Figure 2-1 illustrates two grid patterns usually used in two-dimensional systematic sampling.



(a) (b)

Figure 2-1. Two grid patterns that are usually used in two-dimensional systematic sampling: (a) a square lattice, (b) a triangular lattice.

2.6.1.2 Generalized Random Tessellation Stratified Sampling

The most commonly used spatially balanced sampling method is Generalized Random Tessellation Stratified (GRTS) sampling introduced by Stevens, D. and Olsen (1999, 2003, 2004). GRTS is a generalization of the random-tessellation stratified (RTS) design (Dalenius et al., 1961; Olea, 1984; Overton & Stehman, 1993). The RTS design selects sampling units through a two-step process: at the first step, a regular tessellation coherent with a regular grid is randomly located over the region of interest; and then at the second step, a random unit is selected within each random tessellation cell (Stevens, D. & Olsen, 2004). Since RTS does not allow variable probability spatial sampling, Stevens, D. (1997) introduced the multiple-density, nested, random-tessellation stratified (MD-NRTS) design to provide for variable spatial sampling intensity.

Stevens, D. and Olsen (2004) extended the notion of the MD-NRTS design to a procedure that can potentially create an infinite series of nested grids. This process, that

is used in the GRTS method, results in a function that maps a two-dimensional space into one-dimensional space while attempting to preserve the spatial location of units.

The GRTS method aligns the sampling units which are located within a geographic region (two-dimensional space) using a process termed hierarchical randomization. Then by applying the systematic design, the sampling units are selected, and finally the selected units are mapped back to their original locations. The GRTS method is summarized in the steps below:

1- Map the region of the population of interest into a unit square.

2- Subdivide the unit square into the same size and nested grid cells such that the total inclusion probability for a cell (expected number of samples in the cell) is less than 1. In the first step of the division process, the unit square is subdivided into 4 quadrats (sub-cells) and then each of these is divided into sub-sub-cells, and so on. This process is called hierarchical quadrat partitioning. Figure 2-2 illustrates the first three levels of a hierarchical quadrat partitioning.

3- Use a quadrant-recursive function to order the cells so that two-dimensional proximity relationships are preserved. For this, every time a cell is subdivided, each sub-cell is assigned an address corresponding to the order of subdivision. Each cell address is based on the four numbers {0, 1, 2, 3}. As an example, the address of cross-hatched sub-cell in Figure 2-2 is 213.

   At each level of partitioning, addresses of cells are revised to new ones by randomly permuting the digits. For example, by randomly assigning numbers 2, 3 and 1 to 2 at the first, second and third level of partitioning respectively, and assigning numbers 0, 1 and 2 to 3 at the first, second and third level of portioning respectively, the address of 222 and 323 can be transformed to 231 and 032 respectively. This transformation introduces stochasticity to the sampling design. Without this transformation, the bottom left quadrant and the top right quadrant in Figure 2-2 would always have the address 000 and 333 respectively, and so on.

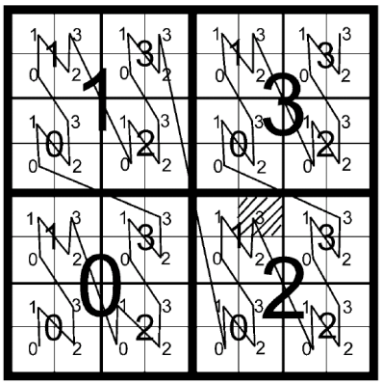


Figure 2-2 the first three levels of a hierarchical quadrat partitioning in GRTS (courtesy of Stevens, D. and Olsen (2004)).

4- Sort the grid cells according to their revised addresses and put them on a real line. In an equal probability sampling, each cell has an equal interval length on the line whereas in unequal probability sampling, each cell is placed at an interval on the line that is proportional to its inclusion probability. For example, if unit grid cell 001 has an inclusion probability which is three times bigger than the inclusion probability of grid cell 000, the length on the real line of grid cell 001 will be three times as long as that for unit B (see Figure 2-3).

5- Select a systematic sample from sampling the line using the Brewer and Hanif (2013) method. For selecting an equal systematic sample of size n from a population with N units, the line is divided into $\frac{N}{n}$ length intervals, selecting a starting point randomly between $(0; \frac{N}{n})$, say k, and then select every $(k + i\frac{N}{n})$ point for $i = 1, \dots, n-1$. If the point occurs within one of the units, then that unit is selected. Figure 2-3 illustrates the implementation of the Brewer and Hanif (2013) method for selecting equal and unequal probability samples.

6- Transform back the addresses of selected grid cells, using the same permutation algorithm but in the reverse direction.

7- Map back the selected grid cells using the original addresses.

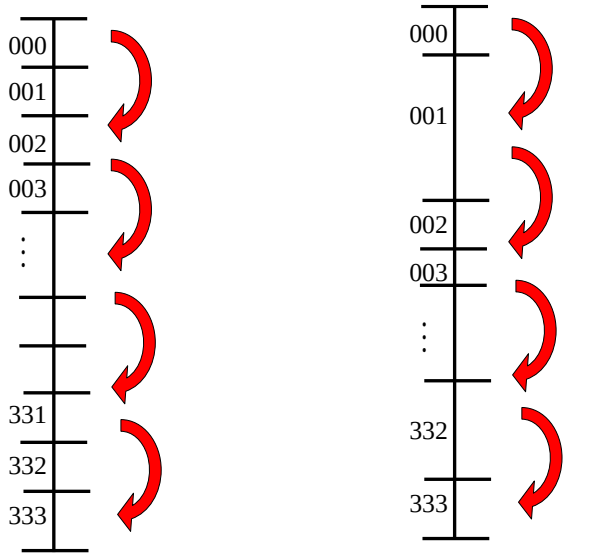


Figure 2-3 implementation of the Brewer and Hanif (2013) method for selecting equal and unequal probability samples.

2.6.1.3 Spatially Correlated Poisson Sampling

A few years after introducing GRTS, Grafström (2012) presented a method called spatially correlated Poisson sampling (SCPS). Grafström's design is based on that proposed by Bondesson and Thorburn (2008), termed correlated Poisson sampling (CPS). CPS is a list sequential PPS sampling method that is suitable for real-time sampling (Meister, 2004). In real time sampling, the units of the population are visited by the sampler one by one in some order. It is a decision to be made by the sampler at the visit whether the unit should be sampled. In real time sampling, there is no probability of revisiting units at a later time. For implementing a list sequential method in a population in which the units are labelled 1 to N (according to the order they are visited or some other order), the sampling outcome (i.e., in terms of including or excluding the unit in the sample) is first decided for the unit labelled 1, then for the unit labelled 2 and so forth. In fact, the sample selection process is done at several steps so that after selecting a unit at each step the inclusion probabilities of the remaining units are updated.

By considering distances between units, rather than just having an ordered list of the units, Grafström (2012) adapted the CPS method to SCPS. By considering distances

between units it aims to create a negative correlation between the inclusion probabilities of units that are close to each other.

In CPS, the first unit is sampled with probability π_1 and if it is included in the sample, $U_1 = 1$, otherwise $U_1 = 0$. When unit $j-1$ has been visited and the value (0 or 1) of the sampling indicator U_{j-1} has been recorded, the inclusion probabilities for all the remaining units are updated. The updated probabilities are denoted $\pi_i^{(j-1)}, i \geq j$ and unit j is then sampled with probability $\pi_j^{(j-1)}$.

Let $\pi_i^{(0)} (= \pi_i)$ be a predefined (initial) inclusion probability of unit i and let $\pi_i^{(j-1)}, i \geq j$ be the updated inclusion probability of unit i when unit $j-1$ has been visited. Let, in the j^{th}outcome, $U_1, U_2, \dots, U_j$ corresponding to the 1^{st} to j^{th} unit respectively have been defined such that $U_k \;\; k = 1, \dots, j$ is equal to 1 if the k^{th}unit was included in sample and 0 otherwise. Having this information, $\pi_i^{(j)}$ can be calculated using Equation (2.14) below:

$$\pi_i^{(j)} = \pi_i^{(j-1)} - \left(U_j - \pi_j^{(j-1)}\right) w_{i-j}^{(i)} \qquad i \geq j+1; j = 1, 2, \dots \qquad (2.14)$$

where $w_{i-j}^{(i)}$are the weights given to units $i = j+1, j+2, \dots, N$ by unit j.

In SCPS, Grafström introduced "maximal weights" and "Gaussian preliminary weights" as two different methods for assigning weights that express the distances between units.

In the maximal weight strategy, in the j^{th} step, the unit j gives the largest weight to the closest unit among the units $i = j+1, \; j+2, \dots, N$. Then, the second closest unit to the j^{th} unit receives the next largest weight, and so on, with the restriction that the sum of the weights equals unity ($\sum \mathrm{w_i} = 1$). For units with equal distances, the weights are distributed equally.

In the Gaussian preliminary weights strategy, some preliminary weights with sum 1 are associated with the units. The preliminary weights are controlled by a Gaussian distribution centred at the position of unit j as below:

$$w_j^{(i)*} \propto exp(-(d(i,j)/\sigma)^2), \qquad i = j+1, j+2, \dots, N \qquad (2.15)$$

where $w_j^{(i)*}$ is the preliminary weight given to unit i by unit j, σ is a parameter that can control the spread of weights and can be chosen according to the distance between units, and $d(i,j)$ is the distance between units i and j. In this strategy, the biggest weight is allocated to the nearest unit, so one option for choosing σ is the average (or median) of the distances between each unit and its closest neighbour (Grafström, 2012).

2.6.1.4 Local Pivotal Methods

In addition to SCPS, Grafström et al. (2012) proposed two other spatial sampling methods – local pivotal methods 1 and 2 (LPM1 and LPM2). These methods are based on the pivotal method (PM) presented by Deville and Tille (1998). In LPMs the population units' inclusion probabilities are updated iteratively until n units have inclusion probabilities equal to 1. The main idea of these methods is to create a negative correlation between the inclusion probabilities of close units. In this way, the probability of selecting adjacent units together in a sample is decreased.

The pivotal method in each step of sample selection modifies the inclusion probabilities of only two units. So, for a population of size N, a sample is obtained by updating the inclusion probabilities in N steps at most. The process of updating continues until the inclusion probabilities of all the units equal either 1 or 0. When the inclusion probability of a unit is updated to either 1 or 0, this unit is "finished" (Grafström et al., 2012).

If π_i and π_j are inclusion probabilities of the i^{th} and j^{th} unit respectively, the PM rule produces $\acute{\pi}_i$ and $\acute{\pi}_j$ as updated inclusion probabilities according to the following rule:

if $\pi_i + \pi_j < 1$, then

$$(\acute{\pi}_i, \acute{\pi}_j) = \begin{cases} (0, \pi_i + \pi_j) & \text{with probability } \dfrac{\pi_j}{\pi_i + \pi_j} \\ (\pi_i + \pi_j, 0) & \text{with probability } \dfrac{\pi_i}{\pi_i + \pi_j} \end{cases}$$

and if $\pi_i + \pi_j \geq 1$, then (2.16)

$$(\acute{\pi}_i, \acute{\pi}_j) = \begin{cases} (1, \pi_i + \pi_j - 1) & \text{with probability } \dfrac{1 - \pi_j}{2 - \pi_i - \pi_j} \\ (\pi_i + \pi_j - 1, 1) & \text{with probability } \dfrac{1 - \pi_i}{2 - \pi_i - \pi_j}. \end{cases}$$

At the first step, at least one of the units is finished. Finished units are not allowed to be chosen in the next step, so the problem of sample selection is reduced to a population with size of at most $N - 1$ units at the second step. Recall that the updating process is repeated until all of the inclusion probabilities are changed to either 0 or 1.

The process of selecting a sample by LPM1 is as follows:

a) Randomly choose one unit i.
b) Choose unit j, a nearest neighbour to i. If two or more units have the same distance to i, then randomly choose one of them with equal probability.
c) If j has i as its nearest neighbour, then update the inclusion probabilities of units i and j according to Equation (2.16). Otherwise go to (a).
d) If all units are finished, then stop. Otherwise go to (a).

The process of selecting a sample by LPM2 is similar to the process of LPM1, but in this method it is not necessary to find out whether unit i is the nearest neighbour of unit j. In LPM2, (c) is removed from the process and the inclusion probabilities of both units i and j are directly updated.

Of the two strategies for selecting sampling units introduced by Grafström et al. (2012), LPM1 produces a more spatially balanced sample, whereas LPM2 is simpler and faster.

In the LPM algorithm, after selecting the unit i randomly, finding unit j, the nearest neighbour to i among the entire population is a computationally intensive process. The expected number of computations needed to select a sample by LPMs is

proportional to N^3 and N^2 for LPM1 and LPM2 respectively (Grafström & Ringvall, 2013); so it can take a long run-time to select a sample from large populations. However, for LPM2, the complexity can be reduced to $O(N\,log(N))$ when *k*-d trees (Bentley, 1975) are used to compute neighbours (Grafström & Lisic, 2016). Hence, it is actually fast (in terms of computational complexity), but this does not necessarily correspond to a fast run time.

Grafström et al. (2014) proposed to expedite the LPM process by restricting the search for unit i's closest neighbour to some smaller local subset instead of the whole population. In order to find that local subset, firstly, the list of the population units is sorted by some auxiliary variables (e.g., spatial coordinates or some other auxiliary variable that is important for the distance). Then, the potential neighbour units are defined among a limited number of h undecided units backwards and forwards from unit i in the list. The length of h is arbitrary but it should not be made too small. One step in implementing this speed optimization, which is called suboptimal LPM, is shown in Figure 2-4. Figure 2-4 illustrates a population with $N = 15$ units that have been ordered according to a relevant variable associated with the distance. Decided units and undecided units are shown by solid squares and white squares, respectively. Assume that unit $i = 7$ is selected randomly. For implementing this method, one can restrict oneself to finding the nearest neighbour to unit i among a subset of undecided units with $h = 3$. The subset includes units $\{2, 3, 6, 8, 9, 10\}$.

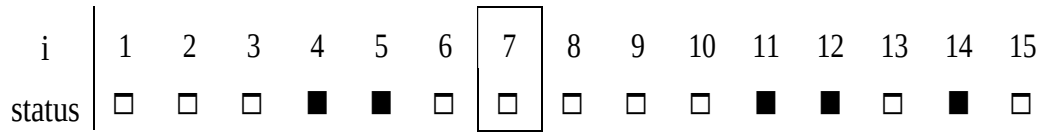


Figure 2-4 An example of implementing the suboptimal LPM in a population with 15 units that have been ordered according to a relevant variable associated with the distance. Solid squares denote decided units and white squares denote undecided units. Unit $i = 7$ is selected randomly. A local subset that contains unit i's potential neighbours is selected among undecided units by considering $h = 3$.

A fast method for implementing this process and a new k-d tree implementation of LPM2 are available in the R package BalancedSampling (Grafström et al., 2014; Grafström & Lisic, 2016).

2.6.1.5 Other Spatially Balanced Sampling

It has been argued that the sample estimates should equal the true known totals of the auxiliarly variables – a property called balanced sampling (Deville & Tillé, 2004; Tillé, 2006, 2011). The CUBE method introduced by Deville and Tillé (2004) is the most commonly used method for selecting a balanced sample.

Although the CUBE method has been introduced in a non-spatial context, by considering the spatial coordinates of the population units as auxiliary variables, the method can be considered a spatial technique (Benedetti et al., 2017).

Grafström and Tillé (2013) also combined the CUBE method and LPM method together and introduced a new spatially balanced sampling method called "doubly balanced sampling" method. A sample selected by this method is well spread over the population and at the same time the Horvitz–Thompson estimators of the auxiliary variables available on all the sampling units are almost equal to their true values in the population.

"Dependent areal units sequential technique" (DUST) (Arbia, 1990, 1993) is another sampling method that avoids the selection of neighbouring regions in an area sampling. This method is a GIS-based sequential technique that works by updating inclusion probabilities of units at each step (Brewer & Hanif, 1983). The procedure of DUST is developed along three steps. In the first step the spatial correlation (β) in a proxy variable (Y) is estimated at various spatial lags (the definition of spatial lags could be found in Haining, 1993). In the second step stationarity of the various order correlations (i.e., $\beta's$**)** is tested. In the third step the spatial correlation of the proxy variable Y is employed to assign weights to the sampling units. If $\beta = 0$ the sampling units are selected by simple random sampling method. If $\beta \neq 0$ the sampling units are selected sequentially by assigning a weight varying at each step. The weights corresponding to the j^{th} sampling unit is $\prod_{i=1}^{j-1}(1-\beta^{d_{ij}}) \quad j = 1, \dots, n$, where n is the sample size and d_{ij} is the distance between units i and j.

Benedetti and Piersimoni (2017) also developed a spatially balanced sampling method that can be used to select a sample of size n in exactly n steps. In each step the selection probability of not–selected units are updated depending on their distance from the units that are already selected in the previous steps. The algorithm starts by

randomly selecting a unit i with equal probability from the population ($U = \{1, 2, \dots, N\}$). Then, at every step where $t \leq n$, the algorithm updates the selection probabilities of every other units of the population according to

$$\pi_j^t = \frac{\pi_j^{t-1} \bar{d}_{ij}}{\sum_{j \in U} \pi_j^{t-1} \bar{d}_{ij}} \tag{2.17}$$

where π_j^{t-1} is the selection probability of the unit j at step $t-1$, and $\bar{d}_{ij}$ is an appropriate transformation applied to the distance matrix ($D_U = \{d_{ij}; i, j = 1, \dots, N\}$). The transformation is considered in order to standardize the distance matrix to have known and fixed products by row $\prod_{i \neq j, i \in U} d_{ij}$ and column $\prod_{i \neq j, i \in U} d_{ij}$.

2.6.2 Parameter Estimation in Spatially Balanced Sampling Methods

In spatially balanced sampling methods, the population total can be estimated by a standard design-based estimator such as the HT estimator given in Equation (2.3). However, the Sen–Yates–Grundy estimator given in Equation (2.5) for estimating the variance of the HT estimator in spatially balanced sampling methods may be unstable because the second order inclusion probabilities of neighbouring units are often zero or near to zero (Robertson et al., 2013). In these cases, Stevens, D. and Olsen (2004) presented an estimator called the "local mean variance" estimator, which is a contrast-based estimator (Yates, 1953; Overton & Stehman, 1993; Wolter, 2007). This estimator was first developed to estimate the variance for the GRTS method, and it has more recently been used to compute the variance estimators for other spatially balanced sampling methods.

The local mean variance estimator is given by:

$$\widehat{\mathrm{V}}_{\mathrm{NBH}}\left(\hat{Y}_T\right) = \sum_{i \in s} \sum_{\mathrm{j} \in \mathrm{D_i}} \mathrm{w_{ij}} \left({}^{\mathrm{y_j}}/_{\pi_j} - \bar{\mathrm{y}}_{\mathrm{D_i}} \right)^2 \tag{2.18}$$

where D_i is a neighbourhood to unit i, containing at least four units, $\bar{y}_{D_i}$ is the total responses of units that are located in the neighbourhood of unit i, and w_{ij} are weights that decrease as the distance between units i and j increase and satisfy $\sum_{j \in D_i} w_{ij} = 1$. More details about computing the weights (w_{ij}) can be found in Stevens, D. and Olsen (2003).

2.6.3 Spatial Coverage

As mentioned earlier, much of the interest in using spatially balanced sampling methods is spreading the sample over the population and avoiding the selection of neighboring units. Spatial balance can be measured and tested in different ways. This section briefly reviews some techniques that will be used in the next chapters to test the spatial coverage of a sample.

2.6.3.1 Spatial Point Pattern Analysis

A spatial point pattern analysis provides statistical methods to study the spatial arrangements of units in the region of the population of interest. Study of spatial point patterns has a long history and its applications appear widely in many different areas of study (Ripley, 1977, Getis, 1984, Upton & Fingleton, 1985). This thesis uses some of the methods in an exploration of spatial point patterns to evaluate the spatial pattern of selected sampling units.

Generally, the spatial point pattern analysis methods are classified into quadrat-based and distance-based methods. Quadrat-based methods are based on overlaying areas of equal size on the region of the population of interest, whereas distance-based methods develop statistics based on the distribution of distances between the sampling and neighbouring units.

The simplest form of quadrat-based methods is the quadrat method where the region of the population of interest is divided into some small quadrats of the same size. Quadrats may have any desired shape, but they are usually square or circular. After counting the frequency of sampling units in each quadrat, a test statistic can be calculated using:

$$T = \frac{(m-1)s^2}{\bar{x}} \tag{2.19}$$

where m is number of quadrats, $\bar{x}$ and s^2 are the observed average and observed variance of the frequency of units among quadrats, respectively. To test the departure from complete spatial randomness, T can be compared to a χ^2 distribution with $m-1$ degrees of freedom.

Quadrat-based methods have some drawbacks when they are used to quantify the spatial features of different samples and designs, because choices of size and shape of quadrats can produce different results (Wong & Lee, 2005). Also, quadrat-based methods are based only on the density of units and do not measure the spatial variations within the quadrats.

In contrast to quadrat-based methods, distance-based methods assume that in most spatial configurations, the existing patterns and similarity among units can be reflected by the distance between them.

Ripley's *K* function introduced by Ripley (1977) and popularized by Kenkel (1988) is a prevalent statistic that describes point patterns over a spatial population. This function is generally based on all the distances between locations of units in the study area and is defined in Equation (2.20):

$$K(h) = \lambda^{-1} E[n_h] \tag{2.20}$$

where n_h is the number of units within distance h of a randomly chosen sampling unit and λ is the density (number per unit area) of units.

There are alternative functions for distance-based methods (such as the G function or the F function), but Ripley's *K* function is useful because it considers the nearest distance, and as such it can describe the concentration of sampling units at a range of distances simultaneously.

Ripley's *K* function for a selected sample can be estimated by constructing a circle of radius r around each sampling unit i and counting the number of other sampling units (j) that fall inside this circle. Let R and n be the area of the region of interest and number of sampling units respectively, and let d_{ij} represent the distance between sampling units i and j. Then, the estimated value of the *K* function for a specific r is calculated by:

$$\widehat{K}(r) = \frac{R}{n^2} \sum_{i} \sum_{\substack{j \\ j \neq i}} \frac{I_r(d_{ij})}{w_{ij}} \tag{2.21}$$

where

$$I_r(d_{ij}) = \begin{cases} 1, & \text{if } d_{ij} \leq r \\ 0, & \text{otherwise'} \end{cases}$$

and w_{ij} is an edge correction. This edge correction is 1 if the whole circle around unit i is located in the region of the population of interest, otherwise it would be considered a proportion of the circumference of the circle that falls inside the region of the population of interest.

Under the assumption of complete spatial randomness, the expected value of $K(r)$ is πr^2. The values of $K(r)$ in a clustered sample is greater than πr^2.

By comparing the observed Ripley's *K* function with the envelope obtained from simulations assuming complete spatial randomness, one can make deductions about the clustering behavior of the point pattern.

2.6.3.2 Voronoi Polygons

Another approach for measuring the spatial balance of a sample introduced by Stevens, D. and Olsen (2004) is based on the concept of Voronoi polygons. Here, a Voronoi polygon consists of all points closer to a particular sampled unit than any other. Figure 2-5 shows the Voronoi polygons generated around sampling units in a given population with 56 units.

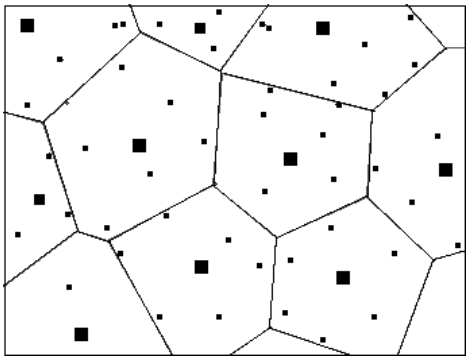


Figure 2-5 The Voronoi polygons generated around sampling units in a given population with 56 units. Selected sampling units are shown enlarged.

The spatial balance of the selected sample of size n is then defined as:

$$\zeta = \frac{1}{n}\sum_{i \in s}(v_i - 1)^2 \qquad (2.22)$$

where v_i indicates the sum of the inclusion probabilities of all units in the Voronoi polygon related to the i^{th} sampling unit.

Lower values of ζ indicate a higher level of spatial balance. However, because the range of ζ is not fixed, it can only be used in a comparative way and cannot determine absence or presence of spatial balance in an individual sample (Tillé et al., 2017). Recently, Tillé et al. (2017) introduced a new index based on *Moran's I* that has a finite range from -1 (perfect spatial balance) to $+1$ (maximum clustered), and can evaluate the degree of spatial balance in a sample.

This thesis uses ζ as it just aims to compare the level of spatial balance among different samples selected from the same population.

2.7 Conclusions

After introducing the concept of probability sampling, this chapter provided a review of the relevant literature on different features of household sampling surveys. Since the application of spatial sampling methods is a new topic in household surveys, the properties of some common spatial methods were reviewed in this chapter. Finally, in the last section of this chapter, some criteria that evaluate the spatial balance of the sample were introduced.

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Chapter 3 *Balanced Acceptance Sampling and its Application to an Intertidal Survey*

3.1 Introduction

In the previous chapter, some common spatially balanced sampling designs were reviewed. In this chapter, another spatial sampling method, balanced acceptance sampling (BAS), introduced by Robertson et al. (2013) is presented.

BAS is relatively new, and there has been a growing interest for its implementation in environmental studies. McDonald, L. et al. (2015) and Keinath and Abernethy (2016) used BAS to select grid cells in different regions of the United States in order to ensure the spatial representativeness of the sample in a study of black-tailed prairie dogs. In another study, Howlin and Mitchell (2016) used BAS to select locations in Bighorn Canyon National Recreation Area in order to monitor bat populations in the area.

In Section 3.2, a thorough background to BAS will first be presented. In Section 3.3, the application of BAS to a case study of crustaceans will be demonstrated and the results of the implementation with a two-dimensional systematic sampling method will be compared. The work described in this chapter has already appeared in a published journal paper (Abi et al., 2017).

3.2 Background to BAS

3.2.1 Random Numbers and Methodology of BAS

Pseudo-random numbers and quasi-random numbers are different types of random numbers which are generally used in sampling theory and simulation studies. In SRS, for instance, a sample is selected using pseudo-random numbers. Although pseudo-random numbers have an advantage of generating random numbers that are independently and identically distributed, they may fail to distribute the numbers evenly over the population. In contrast, quasi-random numbers (also called low

discrepancy sequences) have a high level of uniformity in multidimensional spaces (Levy, G., 2002).

Quasi-random numbers can be generated by a number of different methods. The BAS method utilizes a specific type of quasi-random number called a Halton sequence (Halton, 1960), which is an extension of van der Corput sequences in d dimensions. In technical terms, every integer k can be denoted as a sequence of digits $\lambda_K \lambda_{K-1} \dots \lambda_1 \lambda_0$ in the base of a prime number *p* where:

$$k = \sum_{j=0}^{K} \lambda_j p^j \tag{3.1}$$

In Equation (3.1), $\lambda_j \in \{0, 1, \dots, p-1\}$ and K is a positive integer. Furthermore, a radical inverse function of the integer k, $\phi_p(k)$, can be shown by "$0.\lambda_0 \lambda_1 \dots \lambda_{K-1} \lambda_K$", where:

$$\phi_p(k) = \sum_{j=0}^{K} \frac{\lambda_j}{p^{1+j}} \tag{3.2}$$

For example, $k = 8$ has its representation in base $p = 2$ as 1000, where

$$8 = 0 \times 2^0 + 0 \times 2^1 + 0 \times 2^2 + 1 \times 2^3,$$

and its radical inverse is $\phi_2(8) = 0.0001$.

The sequence $\{\phi_p(k)\}_{k=0}^{\infty}$, with elements in [0,1), is called a van der Corput sequence. The first 10 terms of the van der Corput sequence with bases 2 and 3 are

$$\{\phi_2(k)\}_{k=0}^{9} = \{0, \frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{8}, \frac{5}{8}, \frac{3}{8}, \frac{7}{8}, \frac{1}{16}, \frac{9}{16}\}$$

and

$$\{\phi_3(k)\}_{k=0}^{9} = \{0, \frac{1}{3}, \frac{2}{3}, \frac{1}{9}, \frac{4}{9}, \frac{7}{9}, \frac{2}{9}, \frac{5}{9}, \frac{8}{9}, \frac{1}{27}\}$$

respectively.

The van der Corput sequence with base p can be obtained by partitioning the unit interval, [0,1), with respect to $1/p$, $1/p^2$, $1/p^3$ and so on. The partitioning strategy in the van der Corput sequences generates numbers evenly over the unit interval.

In the above example, converting $\phi_2(8)$ back to the decimal system gives $x_8 = 1/16$ which is the ninth term in the van der Corput sequence with base 2.

The d-dimensional Halton sequence, $\{\tilde{x}_k\}_{k=0}^{\infty}$, with elements in $[0,1)^d$, is a collection of d van der Corput sequences using the first d prime numbers as bases as below:

$$\{\tilde{x}_k\}_{k=0}^{\infty} = \{(\phi_{p1}(k), \phi_{p2}(k), \dots, \phi_{pd}(k))\}_{k=0}^{\infty} \tag{3.3}$$

where p_j is the j^{th} prime number.

As an example, the first 10 terms of the two-dimensional Halton sequence are given by the pairs

$$\{\tilde{x}_k\}_{k=0}^{9} = \{(\phi_2(k), \phi_3(k))\}_{k=0}^{9} = \left\{(0,0), \left(\frac{1}{2}, \frac{1}{3}\right), \left(\frac{1}{4}, \frac{2}{3}\right), \left(\frac{3}{4}, \frac{1}{9}\right), \left(\frac{1}{8}, \frac{4}{9}\right), \left(\frac{5}{8}, \frac{7}{9}\right), \left(\frac{3}{8}, \frac{2}{9}\right), \left(\frac{7}{8}, \frac{5}{9}\right), \left(\frac{1}{16}, \frac{8}{9}\right), \left(\frac{9}{16}, \frac{1}{27}\right)\right\} \tag{3.4}$$

Each pair can be viewed as spatial coordinates of the so-called Halton-points, and can be mapped on to a two-dimensional area. The arrangement of the first 10 Halton points obtained in the example above are shown in Figure 3-1.

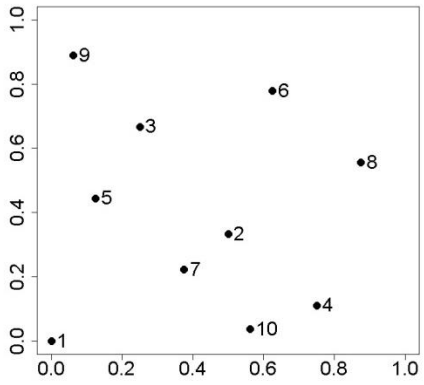

Figure 3-1 The arrangement of the first 10 Halton points with $p_1 = 2$ and $p_2 = 3$.

The Halton sequence is a completely deterministic sequence (Wang, X. & Hickernell, 2000, Robertson et al., 2013). To use it in random sampling, Robertson et al. (2013) propose a random-start Halton sequence, which is created by selecting a random start point for each dimension in the Halton sequence, i.e., by skipping some initial terms of the van der Corput sequences.

3.2.2 Selecting a Sample by BAS

The BAS design employs the Halton sequence and an acceptance/rejection technique to extract a sample with a good spatial coverage. Since population units in spatial studies are often defined by their geographical coordinates, BAS considers each geographical axis as a dimension.

Implementing the BAS method for selecting an equal probability sample of size n in a d-dimensional population could be summarised in two steps as follows:

1- Specify a box that encloses the study region.
2- If the first random-start Halton-point is observed in the study area, it is selected as a sampling unit, otherwise it is discarded and the second Halton-point is checked and so on. This process goes on through the list of the Halton-points until the selected Halton-point is observed in the study area. This process continues until the desired n sampling units have been selected.

By adding an extra dimension that is relevant to the inclusion probability of the population units and applying an acceptance/rejection sampling technique (Neumann, 1951), the BAS method can select an unequal probability sample. To increase the efficiency of the acceptance/rejection sampling, the additional dimension is defined by rescaling the inclusion probabilities for N population units

$$p_i = \frac{\pi_i}{max\,\pi_i} \qquad i = 1, \dots, N \tag{3.5}$$

where π_i and p_i are the inclusion and rescaled inclusion probabilities, respectively.

For selecting an unequal probability sample with BAS in d dimensions, a sufficiently long list of a random-start Halton sequence is generated in $d + 1$ dimensions,

$$\{\tilde{x}_{rk}'\}_{k=0}^{\infty} = \left\{\left(\phi_{p1}(u_1+k), \phi_{p2}(u_2+k), \dots, \phi_{pd}(u_d+k), \phi_{p\,d+1}(u_{d+1}+k)\right)\right\}_{k=0}^{\infty}$$

$$= \left\{\left(\{\tilde{x}_{rk}\}_{k=0}^{\infty}, \phi_{p\,d+1}(u_{d+1}+k)\right)\right\}_{k=0}^{\infty} \quad (3.6)$$

where $u_1, u_2, \dots, u_{d+1}$ are random integers from the uniform distribution on $[0, \mathbb{U}]$, and $\mathbb{U}$ is a sufficiently large integer. If the i^{th} Halton-point in $\{\tilde{x}_{rk}\}_{k=0}^{\infty}$ is located within the study area and $\phi_{d+1}(u_{d+1}+k)$ is smaller than the inclusion probability of the i^{th} unit (i.e., $\phi_{d+1}(u_{d+1}+k) < p_i$), the i^{th} unit is selected.

The algorithm to create a BAS sample is surprisingly straightforward and is available within R (R Core Team, 2017), in the package SDraw (McDonald, T., 2016).

3.2.3 Inclusion Probabilities and Population Estimations

The exact first order inclusion probabilities for units selected with the BAS method, introduced by Robertson et al. (2013), is shown in Equation (3.7):

$$\pi_i = \frac{1}{\mathbb{U}^d} \sum_{j=1}^{\mathbb{U}^d} I(\{x_{rk}^{(j)}\}_{k=1}^{v}) \quad (3.7)$$

where

$$I\left(\left\{x_{rk}^{(j)}\right\}_{k=1}^{v}\right) = \begin{cases} 1, & \text{if unit } i \text{ is selected by the } j^{th} \text{ random} - \text{start Halton sequence} \\ 0, & \text{otherwise} \end{cases}$$

$\mathbb{U}$ is a sufficiently large integer (introduced in Equation (3.6)), and v is the least integer required to select n sampling units.

In fact, there exist $\mathbb{U}^d$ possible random-start Halton sequences that could be created by selecting d random integers between $[0, \mathbb{U}]$; hence, the exact inclusion probability of unit i is expressed as the proportion of these sequences that select unit i.

A simple alternative way to implement BAS, suggested by Robertson et al. (2013), is to select $K \leq \mathbb{U}^d$ random-start Halton sequences from the $\mathbb{U}^d$ possible sequences in random and then randomly select one of them as the sample. In this way, $\mathbb{U}^d$ in Equation (3.7) can be replaced with K, and the inclusion probability of the selected units can be calculated using a Monte-Carlo approximation. Robertson et al. (2013) showed that for large values of K ($K = 10^7$), the Monte-Carlo approximation was good, with $\pi_i \approx {}^{n}/_{N}$ for all i when equal probability sampling was considered.

The inclusion probabilities can be used for population estimation with the HT estimator. However, for the BAS method, as for many other spatially balanced sampling methods, the second order inclusion probabilities for nearby units are zero or near zero. Therefore, the Sen–Yates–Grundy estimator cannot be used as a stable method for estimating the variance of the HT estimator. In this case, a suggestion is to use the local mean variance estimator that Stevens, D. and Olsen (2004) provided for the GRTS method.

3.3 Application of BAS to a Semi-Realistic Dataset

Robertson et al. (2013) compared the spatial balance of the BAS method with other spatially balanced sampling methods by performing simulation studies on some virtual populations. They showed that the selected samples with BAS are spatially balanced and have a competent statistical performance.

In this section the spatial balance and statistical efficiency of the BAS method is evaluated for a crab population in an intertidal marine zone in Qatar. It is compared to two-dimensional systematic sampling (SYS) and simple random sampling (SRS).

3.3.1 Population Description

The data set contains information on crabs from Alkhor, on the east coast of Qatar. In a field study in March 2014, a sample of 80 quadrats was selected. The number of open crab burrows of the species *Nasima dotilliformis* was counted in each 1 m^2 quadrat. The sample was selected by placing 12 parallel strips at equal separations and taking a systematic sample from each strip. The latitude and longitude were recorded for each selected quadrat or sample unit.

This information on quadrat counts and locations was used to create a synthetic population of crabs with the Nadaraya–Watson smoother with Gaussian kernel weighting (E. Nadaraya, 2012; E. A. Nadaraya, 1964; Watson, 1964). This function can be found in the package *spatstat* in R (R Core Team, 2017). Using this package, a population is created that covered 400 × 400 quadrats. Figure 3-2 shows the study area of the generated population counts of *Nasima dotilliformis.*

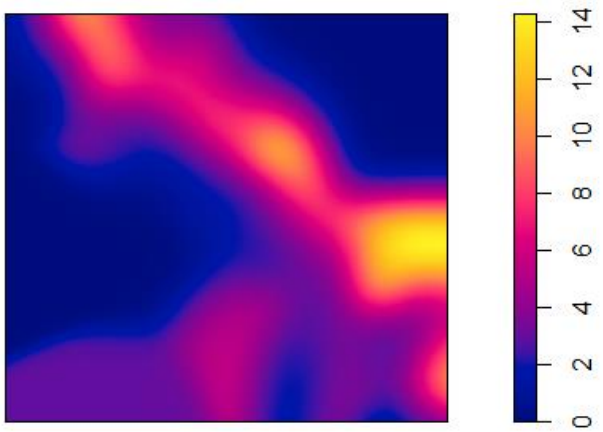

Figure 3-2 Number of Nasima dotilliformis in the simulated population that covers 400×400 equal quadrats.

To detect the spatial autocorrelation among crab burrows across the study area, Moran's I statistic was calculated. The positive calculated value of Moran's I (= 0.9) shows that the quadrats tended to be surrounded by neighbours with similar counts of crab burrows.

The Moran scatter plot (Anselin, 1995) is also a visualisation tool to find how spatially autocorrelated a variable is. The *x*-axis in a Moran scatter plot represents the values of the variable of interest and the *y*-axis shows the mean values of the variable of interest among neighbours of a unit of interest. Neighbourhood units are defined according to the spatial weights matrix that is specified in Moran's I Index.

Considering the rook definition for defining neighbourhood quadrats, the Moran scatter plot of the number of crab burrows in the population under study is illustrated in Figure 3-3.

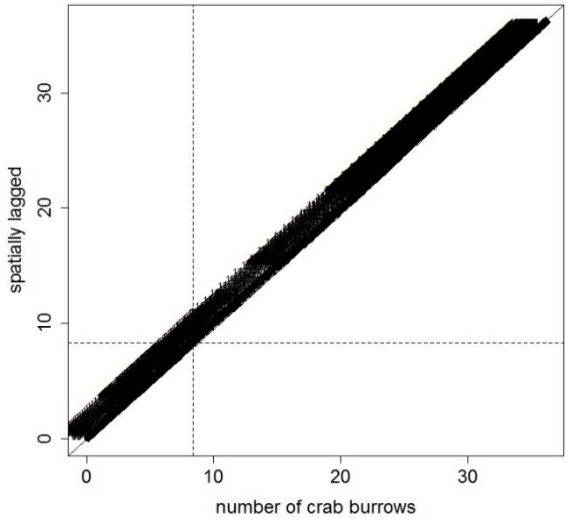

Figure 3-3 Moran scatter plot of number of crab burrows in the study area formed by 400 × 400 quadrats.

The Moran scatter plot is divided into four quadrants. These quadrants are referred to as high-high, low-low, low-high and high-low, relative to the average number of crab burrows, which is shown by the dashed lines in Figure 3-3. The upper right quadrant represents the spatial correlation of quadrats with high average number of crab burrows around neighbours which have also high average number of crab burrows. The upper left quadrant represents the spatial correlation of quadrats with low average number of crab burrows around neighbours that have high average number of crab burrows. The lower left quadrant represents the spatial correlation of quadrats with low average number of crab burrows around neighbours that also have a low average number of crab burrows. The lower right quadrant represents the spatial correlation of quadrats with high average number of crab burrows around neighbours that have a low average number of crab burrows. Figure 3-3 shows that there is a high positive spatial autocorrelation between quadrats in the study area; in fact, the population is highly clustered.

3.3.2 Sample Selection

To implement the BAS method, the study area was scaled to fit in the unit box. A BAS sample of n points was then drawn from the scaled study area.

To carry out two-dimensional systematic sampling, the population was partitioned into equal subregions according to the desired sample size. For example, in the case with a sample size of n = 36, the longitude and longitude were partitioned into 6 equal intervals, and 36 equal subregions were created. The first quadrat was randomly selected from the first subregion. Other quadrats in the remaining subregions had the same position as the first sampling unit within the subregions.

3.3.3 Spatial Coverage and Parameter Estimation

As mentioned in Section 2.6.3, the variance of the sum of the inclusion probabilities of the Voronoi polygons defined by the sample ($\zeta = Var(v_i), \quad i = 1, 2, \dots, n$) can be used to measure how well spread out a sample is. In this study, for each sample size, the process of selecting a sample was repeated 1000 times, and ζ was calculated for each. To compare ζ among different sampling schemes, the average of ζ among all 1000 replications was calculated using Equation (3.8):

$$\hat{\mu}(\zeta) = \frac{1}{1000} \sum_{r=1}^{1000} \zeta_r \tag{3.8}$$

where ζ_r is the ζ of the r^{th} iteration. Small $\hat{\mu}(\zeta)$ indicates good spatial balance.

In this study, the total number of crab burrows in the study area is the parameter of interest and the HT estimator is used to estimate it. The average and mean-square error (MSE) of the HT estimator calculated from 1000 simulated samples were compared for three different sampling schemes. The mean and MSE of the HT estimator for 1000 simulated samples were estimated with:

$$\hat{\mu}\left(\hat{Y}_{HT}\right) = \frac{1}{1000} \sum_{r=1}^{1000} \hat{Y}_{HT_r} \tag{3.9}$$

and

$$\widehat{Var_{SIM}}\left(\hat{Y}_{HT}\right) = \frac{1}{1000} \sum_{r=1}^{1000} \left(\hat{Y}_{HT_r} - Y\right)^2 \tag{3.10}$$

respectively, where $\hat{Y}_{HT_r}$ is the total number of crab burrows estimated from the r^{th} iteration and Y is the true total number of crab burrows in the study area. In this study the value of Y is known and equal to 1,336,781 crab burrows.

The $\hat{\mu}(\zeta)$ for BAS and SYS in comparison to SRS are shown in the third column of Table 3-1. The values of $\hat{\mu}(\zeta)$ corresponding to the BAS and SYS are smaller than values of $\hat{\mu}(\zeta)$ for SRS, which shows that both methods provide more spatially balanced samples than SRS. Systematic sampling has the most uniform spread as shown in Figure 3-4c. Its average $\hat{\mu}(\zeta)$ is very small because each Voronoi polygon is a box and most of these boxes are the same size. Figure 3-4a shows how a sample from BAS is more evenly spread than the sample from SRS (Figure 3-4c).

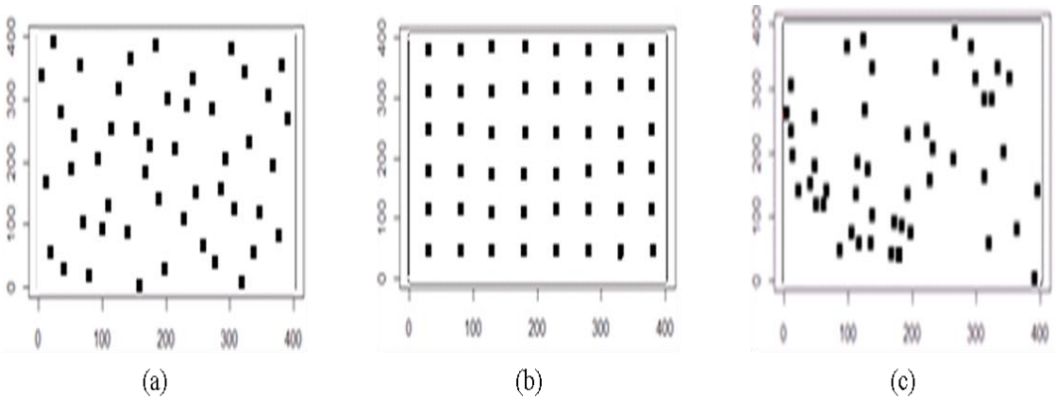

Figure 3-4 A sample of size equal to 48 quadrats drawn using (a) the BAS method, (b) the two-dimensional systematic sampling method, and (c) the simple random sampling method, respectively.

In this study, the estimated variance of each sample selected by BAS and SYS was also calculated using the local mean variance estimator (Equation (2.18)). For each sample size, the average of the estimated variances among 1000 samples ($\widehat{\mathrm{Var}}_{est}$) was calculated by:

$$\widehat{\mathrm{Var}}_{est}(\hat{\mathrm{Y}}_{\mathrm{HT}}) = \frac{1}{1000}\sum_{r=1}^{1000}\hat{\mathrm{V}}_{\mathrm{NBH-r}}(\hat{Y}_T), \tag{3.11}$$

where $\hat{\mathrm{V}}_{\mathrm{NBH-r}}(\hat{Y}_T)$ is the local mean variance that was estimated from the r^{th} sample. The $\widehat{\mathrm{Var}}_{est}(\hat{\mathrm{Y}}_{\mathrm{HT}})$ for the different sampling schemes with the different sample sizes are shown in the last column of Table 3-1.

Table 3-1 $\hat{\mu}(\zeta)$, the simulated variance of the HT estimator and the estimated variance $\widehat{Var}_{est}$ for two sampling schemes with different sample sizes.

n	design	$\hat{\mu}(\zeta)_{complex}/\hat{\mu}(\zeta)_{SRS}$	$\widehat{\mathrm{Var}}_{SIM-complex}/\widehat{\mathrm{Var}}_{SIM-SRS}$	$\widehat{\mathrm{Var}}_{est}$
36	BAS	0.2	0.40	193×10^8
	SYS	0.1×10^{-3}	0.39	168×10^8
81	BAS	0.2	0.22	41×10^8
	SYS	0.1×10^{-3}	0.18	34×10^8
121	BAS	0.2	0.18	22×10^8
	SYS	0.3×10^{-3}	0.22	29×10^8
169	BAS	0.2	0.13	11×10^8
	SYS	0.1×10^{-3}	0.17	17×10^8
196	BAS	0.2	0.12	9×10^8
	SYS	0.1×10^{-3}	0.13	9×10^8
256	BAS	0.2	0.11	5×10^8
	SYS	0.3×10^{-3}	0.14	7×10^8
289	BAS	0.2	0.07	4×10^8
	SYS	0.1×10^{-3}	0.10	6×10^8

The trends of the ($\widehat{\mathrm{Var}}_{complex}/\widehat{\mathrm{Var}}_{SRS}$) for the different sampling methods (BAS and SYS) and sample sizes are shown in Figure 3-5. These two designs had similar estimated variances, with BAS being the more precise than the two-dimensional systematic sampling except for the smaller sample sizes.

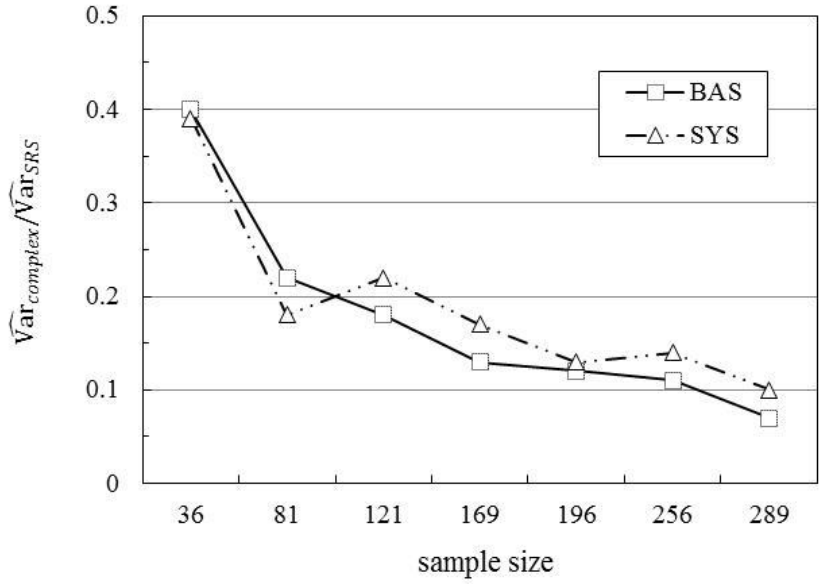

Figure 3-5 The $\widehat{Var}_{complex}/\widehat{Var}_{SRS}$ of two different sampling methods (BAS and SYS) with different sample sizes.

3.4 Further Discussions about BAS

Of the three designs used in this study, BAS and two-dimensional systematic sampling were superior to simple random sampling in terms of spatial spread and precision. In addition to these statistical advantages, there are a number of practical considerations. Encountering unforeseen factors is a common issue in implementing sampling methods on environmental fields; therefore, designing a flexible method that could adapt to field changes is desirable.

Generally, in a two-dimensional systematic sampling method, the quadrats are selected with a fixed distance between quadrats and with a regular pattern. Full coverage of the study area will only be met once the sampling process is completed. In some field situations, completing the entire sampling process may not be possible, for example, if bad weather stops the field surveys early. In this situation, two-dimensional systematic sampling may not have a consistent spread of the quadrats over the study area and there may be gaps where quadrats are not visited. BAS, on the other hand, is able to cover the study area even when sampling is stopped early if quadrats are visited in the order they were generated. For more clarity, assume that because of an extraordinary event we are forced to stop the sampling process at 30 quadrats instead of 48. Figure 3-6 shows the quadrats that will be visited by (a) BAS and (b) two-dimensional systematic sampling if the site ordering is strictly followed.

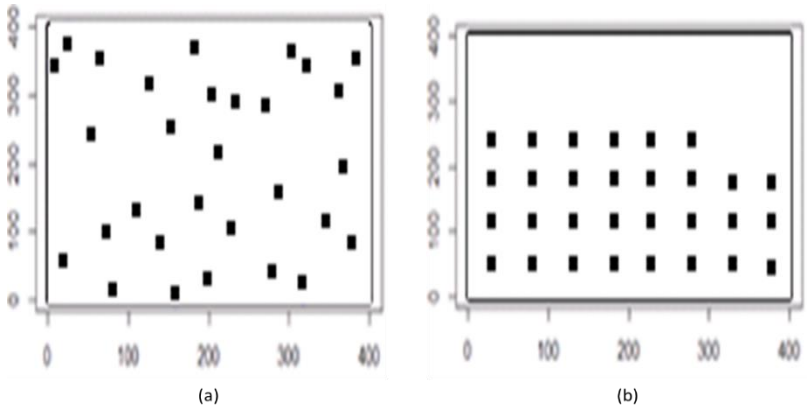

Figure 3-6 The resultant survey when only 30 quadrats instead of 48 quadrats are selected with (a) BAS and (b) two-dimensional systematic sampling method, respectively.

There is still good coverage of the study area by the BAS method if the quadrats are visited in sequential order, whereas the two-dimensional systematic sampling method

is not able to provide coverage over the study area when the sampling process is stopped.

In practice, visiting BAS quadrats in their exact order would be very time-consuming and costly because the study area would be traversed many times. Instead we recommend sets of quadrats are visited. For example, a set could be 6 quadrats (quadrats 1–6, quadrats 7–12, and so on), where the time taken to survey a set relates to a practical unit in time (a half-day for example). In this way, as long as a set is completed there will always be site coverage.

Other practical advantages of the BAS method over the two-dimensional systematic sampling method are as follows:

i. In environmental samples, access to some sampling units may be denied or be impossible. These sampling units are considered as missing values. With BAS, new sampling units can easily be substituted for the missing sampling units. The new sampling unit is added to the sample by continuing the sampling selection process. With the two-dimensional systematic sampling method, adding new sampling units to substitute for inaccessible sampling units may lead to a loss of spatial balance, especially if there are a considerable number of inaccessible sampling units. Note that the missing values should be taken into account in the estimation process.

ii. In situations where there is a change to the sample size during the survey, e.g., extra resources are allocated to the study, spatial spread can be achieved with BAS by continuing the sampling selection process. In contrast, with the two-dimensional sampling method it is more difficult to add extra sampling units without disrupting the regular pattern (Stehman, 2009), unless the count of the extra units is a multiple of the sample size.

iii. In implementing the two-dimensional systematic sampling method, the study area is partitioned into subregions and then sampling units are selected from each of them. When the study area does not have a regular shape, some sampling units may be missed. Figure 3-7 shows the sampling units that are selected by a two-dimensional systematic sampling method

from a study area with irregular shape (the study area is shown in grey). The area within which the study area sits is divided into 9 subregions. In Figure 3-7a, the resultant sample size is only $n = 5$ because many sampling units are outside the study area. With such a predetermined pattern for selecting sampling units, the actual sample size is variable. Consider when the grid is overlaid in a different orientation and different units are selected in the subregions. The resultant sample size in Figure 3-7b is now $n = 3$.

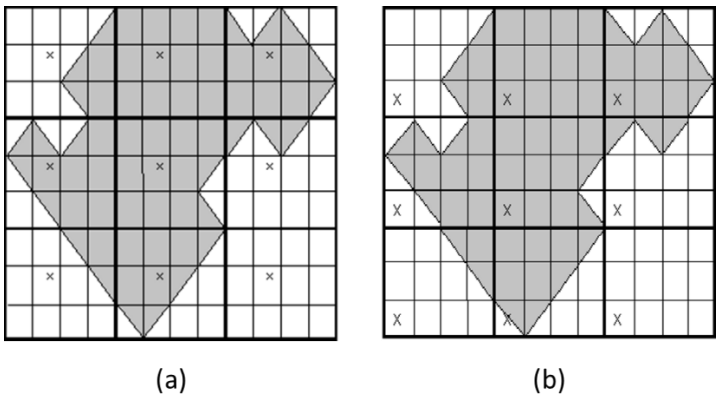

Figure 3-7 Examples of sampling units selected using the two-dimensional systematic sampling method from a study area with irregular shape: (a) the resultant sample size = 5, (b) the resultant sample size =3.

BAS on the other hand, uses an acceptance/rejection sampling technique and discards the selected sampling units that are located outside the study area, and the sample size is predetermined and fixed.

iv. Other benefits of BAS are that sampling units can be selected with unequal selection probabilities. In survey sampling, using auxiliary variables to determine unequal selection probabilities for sample units can be very helpful, for example, to increase sample effort in favourable habitats and to decrease it in unfavourable habitats.

3.5 Conclusions

The BAS method can be used for selecting a sample that is well spread out over the population. In this chapter, after a review of the BAS method, it was used for selecting quadrats to estimate the size of a crab population, and the results were compared to the

results for a two-dimensional systematic sampling and the simple random sampling method. Although the results for BAS and SYS were comparable, BAS has several practical advantages, including its ability to adapt to unexpected changes in the sampling process, which make it preferable.

3.6 References

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Chapter 4 *Population Characteristics and Performance of Balanced Acceptance Sampling*

4.1 Introduction

It is important to consider the characteristics of the population of interest during the design of a sampling scheme. For instance, the similarity between neighbouring units is one of the characteristics of most spatial populations and by using spatially balanced sampling methods selection of neighbouring units in a sample can be avoided. Although there are a number of different spatially balanced sampling methods in the literature, their implementations have been usually for populations where the response variables follow a continuous distribution. In contrast, there has been little discussion on application of spatially balanced sampling methods in the cases where the response variables are dependent binary data (e.g., the presence or absence of a characteristic).

Collecting and analysing the binary data (dichotomous responses) such as presence or absence of children, retirees, disabled persons in the household, whether or not the household possesses a car or has access to running water is a common task in many social surveys. Therefore, it is helpful to consider whether using spatially balanced sampling methods can be useful in these situations. The first part of this chapter intends to examine whether the balanced acceptance sampling (BAS) method, as an example of a spatially balanced sampling method, is an efficient design when the response variable is binary.

The presence (existence) of some specific subgroups in a population is better dealt with via the use of stratified sampling methods. Employing a stratified sampling method ensures that the selected sample includes representation from each subgroup (Cochran, 1977). Whilst stratified sampling is an effective sampling scheme, it can only be implemented when the boundaries of the strata are clearly delineated. Sometimes defining mutually exclusive strata may be a challenging process, and therefore statisticians try to find an appropriate solution to avoid defining strata. With increasing

interest in applying spatially balanced sampling methods, an important question is whether these sampling methods can be used as an alternative to the stratified sampling.

The second part of this chapter seeks to investigate the suitability of applying BAS in stratified sampling. It also compares the efficiency of two possible ways of applying BAS on a stratified population.

4.2 Application of BAS on Populations With Different Spatial Autocorrelation

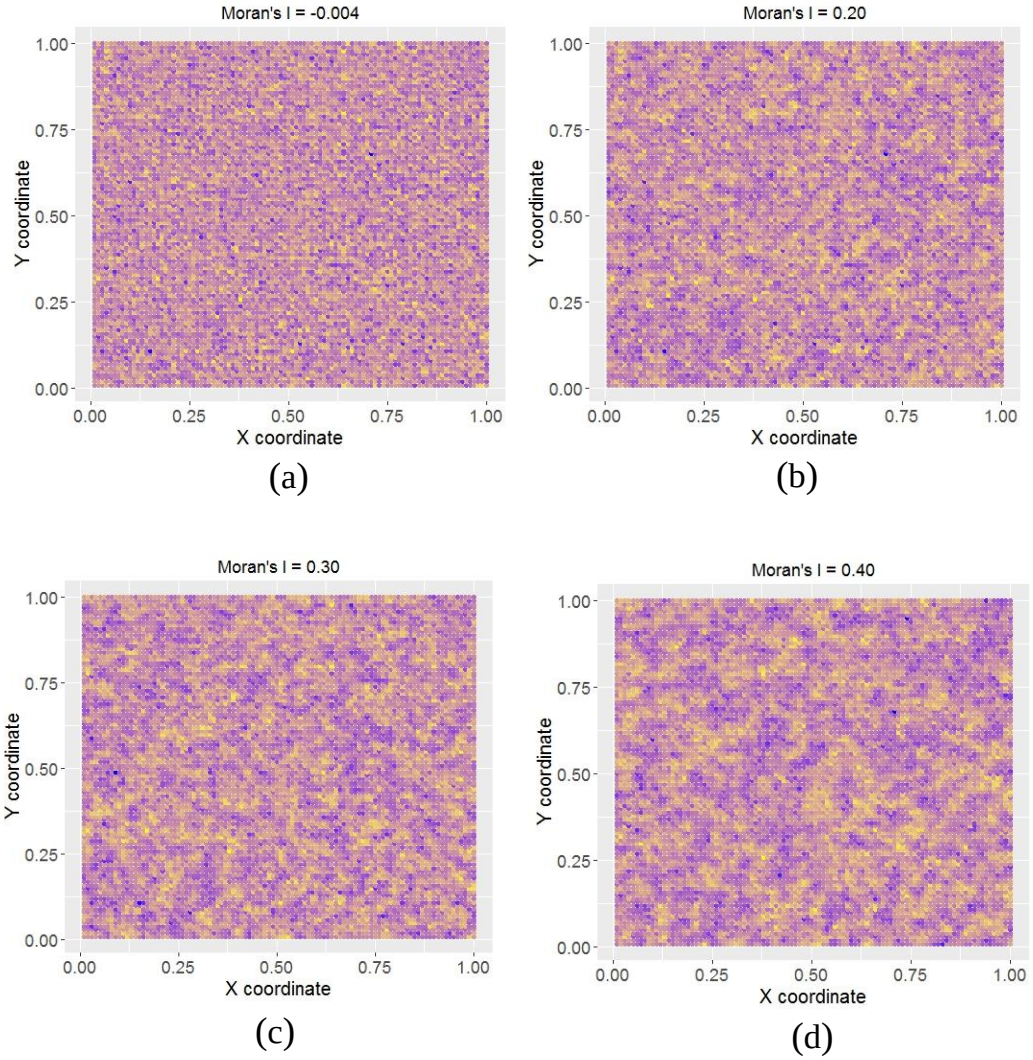
Obtaining precise population estimates is one of the most important goals in selecting a suitable sampling design. Most of the literature in the area of spatially balanced sampling methods have emphasized that these sampling designs are more precise than the non-spatially balanced sampling methods when there is spatial autocorrelation in the population of interest (Stevens, D. & Olsen, 2004; Grafström et al., 2012). In this section, we examine to what extent spatial autocorrelation affects the efficiency of BAS.

The performance of the BAS method in populations with different levels of spatial autocorrelation was examined through conducting simulation studies on synthetic georeferenced populations. Each georeferenced population consisted of 100 by 100 square units, $N = 10{,}000$. From each georeferenced population, 1000 samples were selected using two different sampling methods (SRS and BAS), and the results were compared. To ensure that the results were not affected by the size of the selected samples, the sample selection process was repeated for seven different sample sizes ($n = 50$, 100, 150, 200, 250, 300 and 350). In this evaluation, two types of georeferenced populations were considered: a population where the response has a Gaussian distribution, and a population with binary responses.

4.2.1 Using BAS in Populations Where Observations Have a Gaussian Distribution

In this case, it was assumed that the response values came from a standard normal distribution, $N(0,1)$. After generating populations consisting of 10,000 observations (100 by 100 grid), the simulation study was conducted with different variations of the spatial features. To ensure that the results were consistent with the structure of the

generated populations, 1000 synthetic populations were generated for each spatial feature. In this study, "geoR" package in R (R Core Team, 2017) was used to create different structures of the population, generated with different levels of Moran's I from complete randomness (Moran's I = -0.06) to completely clustered (Moran's I = 0.80). Figure 4-1 illustrates the spatial features of the some of the generated populations with different Moran's I.



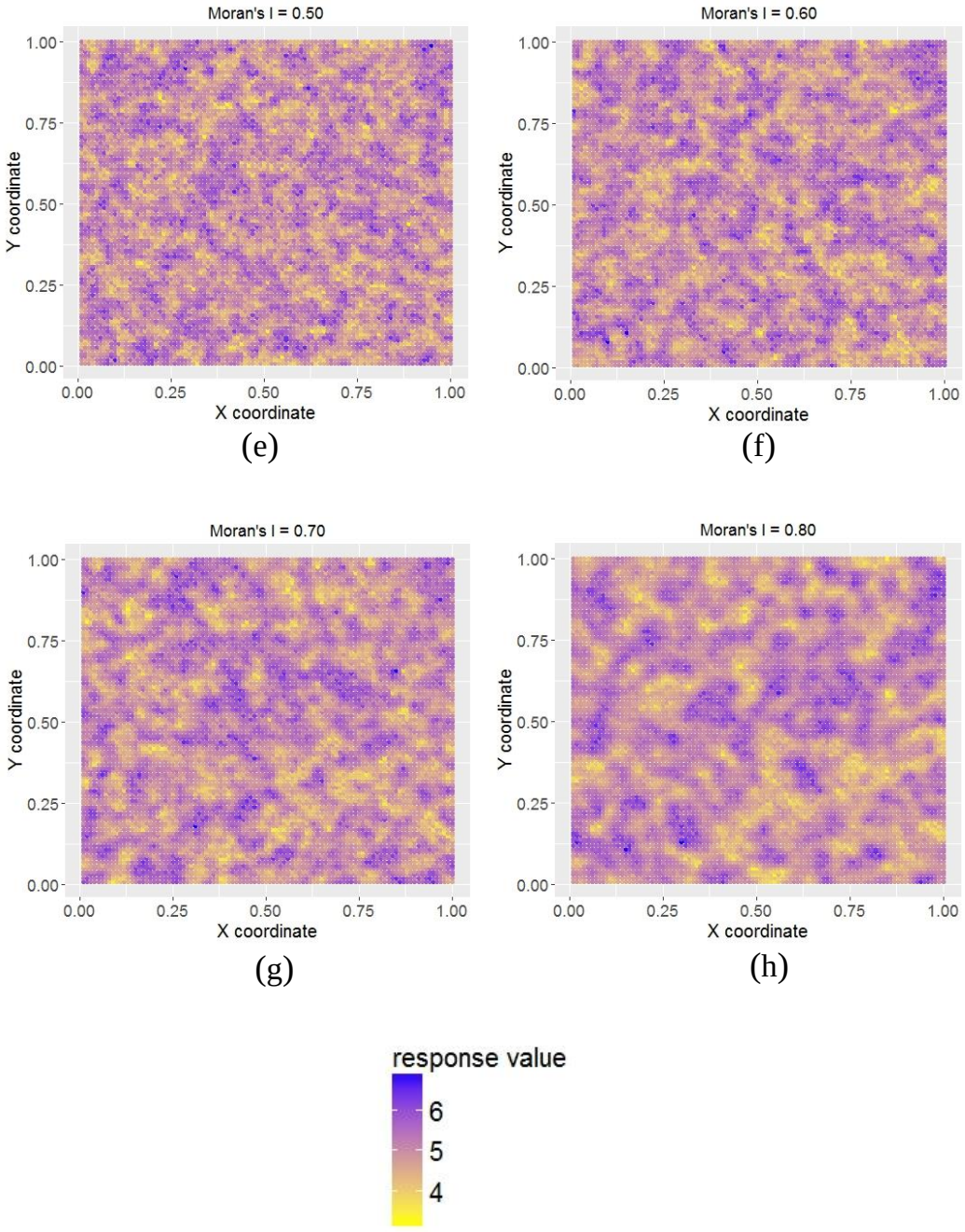


Figure 4-1 The spatial features of the generated Gaussian population with different Moran's I indices.

From each spatial feature of the generated populations, 1000 populations of size 10,000 were generated and for each population, 1000 samples were selected for each considered sample size. This gave a total of 1,000,000 samples for each spatial feature and each sample size. For each sample, the Horvitz–Thompson (HT) estimator of total (Horvitz & Thompson, 1952) was used to estimate the total value of the variable of

interest. After completing the sample selection process, the variance of the HT estimator was estimated using:

$$\widehat{Var}\left(\hat{Y}_{HT}\right) = \frac{1}{1000}\sum_{p=1}^{1000} \hat{Y}_{HT_p}, \tag{4.1}$$

$$\hat{Y}_{HT_p} = \frac{1}{1000}\sum_{r=1}^{1000}\left(\hat{Y}_{rp} - Y_p\right)^2,$$

where $\hat{Y}_{rp}$ is the population total estimated from the r^{th} sample in the p^{th} generated population and Y_p is the true population total in the p^{th} generated population.

For each sample size, the simulated variance of the HT estimator related to the two sampling methods[1] and different spatial features are reported in Table 4-1 and Figure 4-2. The ratio of variance of the HT estimator of the BAS method to the variance of the HT estimator of the SRS, $r_{BAS/SRS}$, for each sample size is also included in Table 4-1 and the relevant graphs are shown in Figure 4-3.

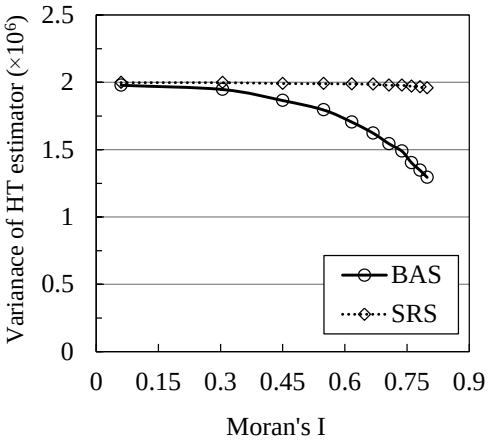
Table 4-1 Simulated variance of the HT estimator when BAS and SRS are employed to select samples (of sizes $n = 50, 100, 150, 200, 250, 300$ and 350) from Gaussian populations with different levels of Moran's I.

Simulated variance of the HT estimator ($\times 10^6$)								
Moran' s I	Design	Sample size						
		50	100	150	200	250	300	350
0.06	BAS	1.98	0.99	0.65	0.48	0.38	0.31	0.27
	SRS	2.00	1.00	0.67	0.50	0.40	0.33	0.29
	$r_{BAS/SRS}$	0.99	0.99	0.97	0.96	0.96	0.95	0.94
0.31	BAS	1.95	0.95	0.61	0.45	0.35	0.29	0.25
	SRS	2.00	1.00	0.67	0.50	0.40	0.33	0.29
	$r_{BAS/SRS}$	0.97	0.95	0.92	0.91	0.89	0.87	0.86
0.45	BAS	1.86	0.89	0.57	0.41	0.32	0.26	0.22
	SRS	1.99	1.00	0.66	0.50	0.40	0.33	0.28
	$r_{BAS/SRS}$	0.94	0.89	0.85	0.83	0.80	0.77	0.76

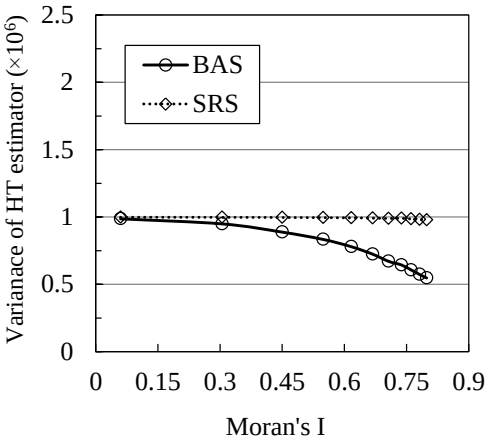
[1] For SRS, the variances are calculated using the theoretical formula (Equation 2.10).

Table 4-1 continued

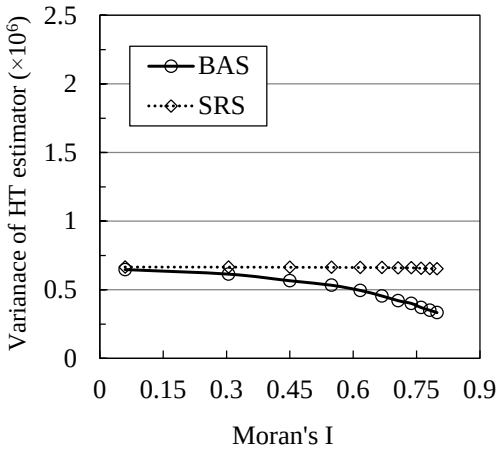
Simulated variance of the HT estimator ($\times 10^6$)								
Moran' s I	Design	Sample size						
		50	100	150	200	250	300	350
0.55	BAS	1.79	0.83	0.53	0.38	0.30	0.24	0.20
	SRS	1.99	1.00	0.66	0.50	0.40	0.33	0.28
	$r_{BAS/SRS}$	0.90	0.84	0.80	0.76	0.74	0.71	0.70
0.62	BAS	1.70	0.78	0.50	0.35	0.27	0.21	0.18
	SRS	1.99	0.99	0.66	0.50	0.40	0.33	0.28
	$r_{BAS/SRS}$	0.86	0.78	0.75	0.71	0.68	0.64	0.62
0.67	BAS	1.62	0.72	0.45	0.33	0.25	0.20	0.16
	SRS	1.99	0.99	0.66	0.50	0.40	0.33	0.28
	$r_{BAS/SRS}$	0.82	0.73	0.69	0.66	0.63	0.60	0.57
0.71	BAS	1.54	0.67	0.42	0.30	0.23	0.18	0.15
	SRS	1.98	0.99	0.66	0.49	0.40	0.33	0.28
	$r_{BAS/SRS}$	0.78	0.68	0.64	0.61	0.57	0.54	0.52
0.74	BAS	1.49	0.64	0.40	0.28	0.21	0.17	0.14
	SRS	1.98	0.99	0.66	0.50	0.40	0.33	0.28
	$r_{BAS/SRS}$	0.75	0.65	0.61	0.57	0.54	0.51	0.48
0.76	BAS	1.40	0.61	0.37	0.26	0.20	0.15	0.13
	SRS	1.97	0.99	0.66	0.49	0.39	0.33	0.28
	$r_{BAS/SRS}$	0.71	0.62	0.56	0.53	0.50	0.47	0.45
0.78	BAS	1.35	0.57	0.35	0.25	0.19	0.15	0.12
	SRS	1.97	0.98	0.65	0.49	0.39	0.33	0.28
	$r_{BAS/SRS}$	0.69	0.58	0.54	0.51	0.47	0.44	0.42
0.80	BAS	1.29	0.55	0.33	0.24	0.18	0.14	0.11
	SRS	1.96	0.98	0.65	0.49	0.39	0.33	0.28
	$r_{BAS/SRS}$	0.66	0.56	0.51	0.49	0.45	0.42	0.41


Varianace of HT estimator (×10^6)
2.5
2
1.5
1
0.5
0
0 0.15 0.3 0.45 0.6 0.75 0.9
Moran's I
BAS
SRS

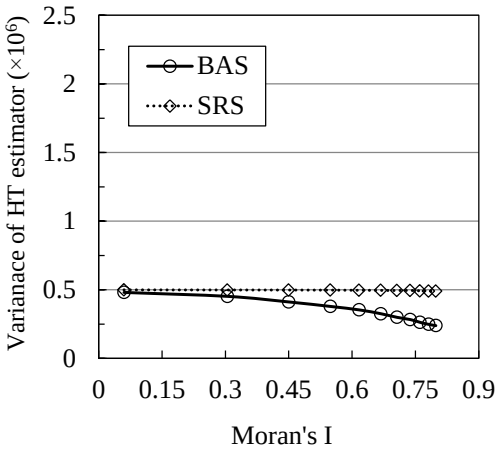
(a)


Varianace of HT estimator (×10^6)
2.5
2
1.5
1
0.5
0
0 0.15 0.3 0.45 0.6 0.75 0.9
Moran's I
BAS
SRS

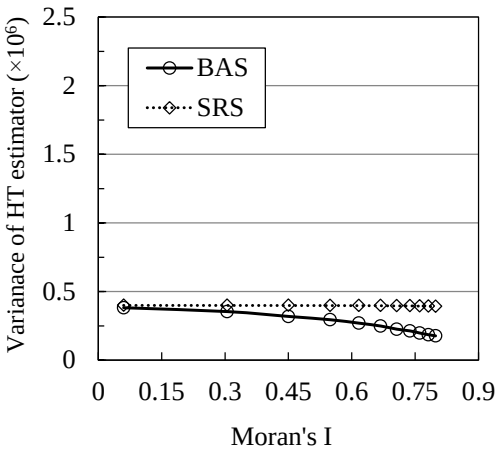
(b)


Varianace of HT estimator (×10^6)
2.5
2
1.5
1
0.5
0
0 0.15 0.3 0.45 0.6 0.75 0.9
Moran's I
BAS
SRS

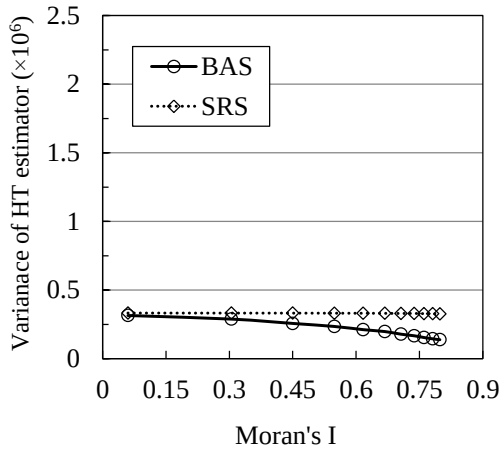
(c)


Varianace of HT estimator (×10^6)
2.5
2
1.5
1
0.5
0
0 0.15 0.3 0.45 0.6 0.75 0.9
Moran's I
BAS
SRS

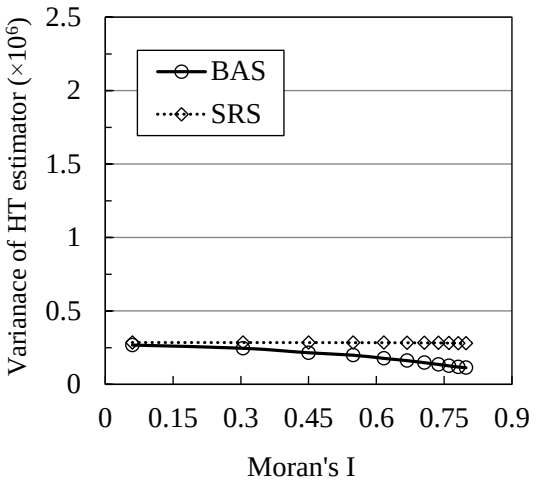
(d)


Varianace of HT estimator (×10^6)
2.5
2
1.5
1
0.5
0
0 0.15 0.3 0.45 0.6 0.75 0.9
Moran's I
BAS
SRS

(e)


Varianace of HT estimator (×10^6)
2.5
2
1.5
1
0.5
0
0 0.15 0.3 0.45 0.6 0.75 0.9
Moran's I
BAS
SRS

(f)



(g)

Figure 4-2 Trend of simulated variance of the HT estimator for Gaussian populations amongst different levels of Moran's I when BAS and SRS are used to select different sample sizes (a) $n = 50$*, (b)* $n = 100$*, (c)* $n = 150$*, (d)* $n = 200$*, (e)* $n = 250$*, (f)* $n = 300$ *and (g)* $n = 350$*.*

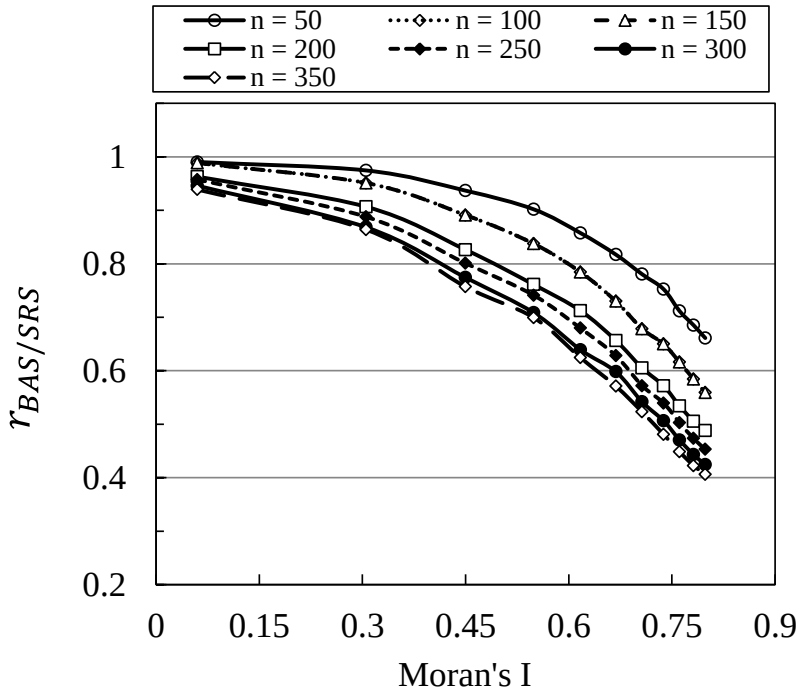
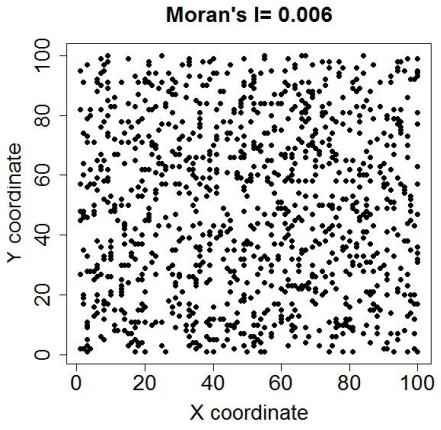


Figure 4-3 The ratio of variance of the HT estimator of the BAS method to the variance of the HT estimator of SRS, $r_{BAS/SRS}$*, for Gaussian populations amongst different levels of Moran's I.*

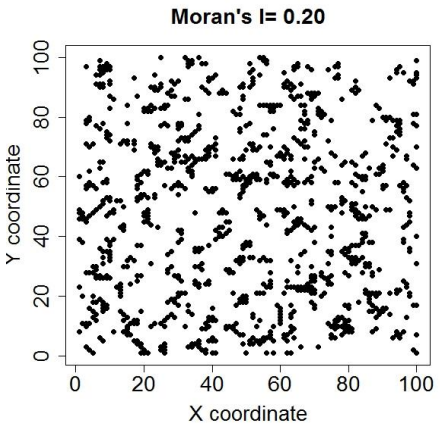
As can be seen in Table 4-1, Figure 4-2 and Figure 4-3, for increasing spatial autocorrelation, the BAS method is more efficient than SRS for all sample sizes. The variance of the HT estimator with SRS is almost constant for different levels of Moran's I, and there is no relationship between the underlying population's spatial pattern and the precision of the estimate. In contrast, there is a significant gap between the simulated variance of the HT estimator in the population with a low Moran's I value and in the population with a high Moran's I value, when BAS is used for selecting the samples. This emphasizes how using BAS for a spatially autocorrelated population with Gaussian response can increase the precision of the estimates.

4.2.2 Using BAS in Populations Where Responses are Binary Data

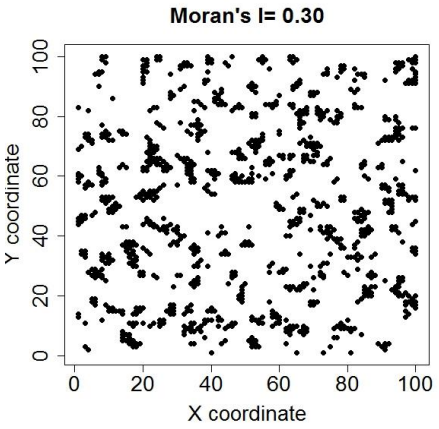
Populations that can be characterised by a Bernoulli distribution where its units have binary responses, such as employment status (employed/unemployed). In order to understand whether the BAS method can be an efficient sampling design to select samples from this kind of population, a simulation study similar to that performed in the previous subsection was conducted. The difference here is that instead of using a normal distribution, the observations were generated from a Bernoulli response distribution. Using "geoR" package in R (R Core Team, 2017), different spatial structures of a population with a Bernoulli distribution with parameter $p = 0.5$ from complete spatial randomness (Moran's I = 0.006) to spatial clustered (Moran's I = 0.80) were generated. Some discussions on generating spatially auto-correlated data can be found in Appendix A. Considered spatial features of some of the generated Bernoulli populations with $p = 0.5$ are illustrated in Figure 4-4.


Moran's I= 0.006
Y coordinate
X coordinate
0
20
40
60
80
100

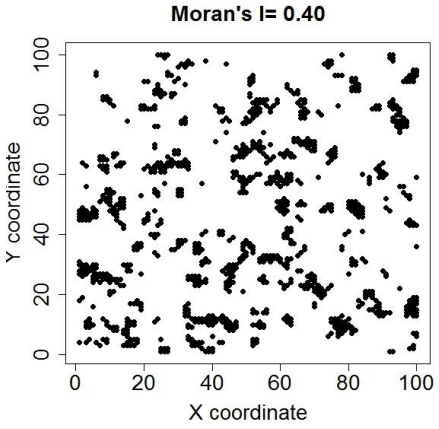
(a)


Moran's I= 0.20
Y coordinate
X coordinate
0
20
40
60
80
100

(b)


Moran's I= 0.30
Y coordinate
X coordinate
0
20
40
60
80
100

(c)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X coordinate
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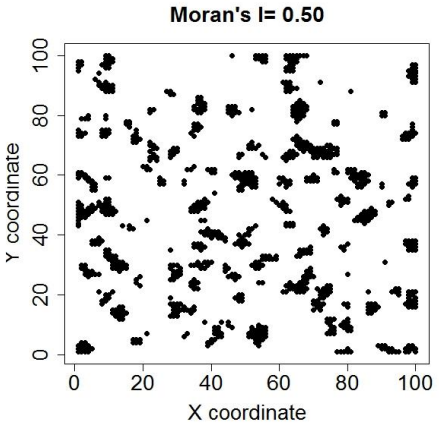
20
40
60
80
100

(d)


Moran's I= 0.50
Y coordinate
X coordinate
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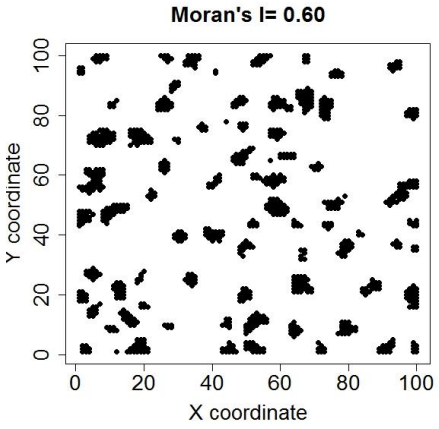
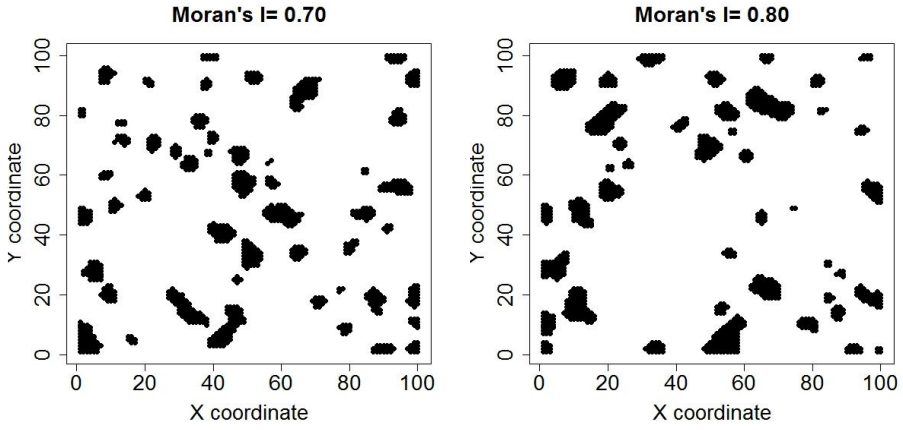
20
40
60
80
100

(e)


Moran's I= 0.60
Y coordinate
X coordinate
0
20
40
60
80
100

(f)



(g) (h)

Figure 4-4 Spatial features of the generated population with a Bernoulli distribution with parameter $p = 0.5$.

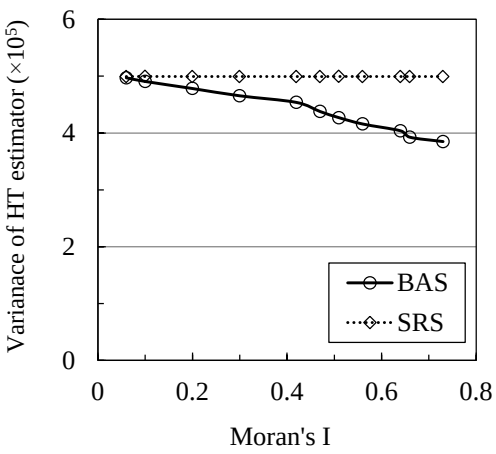
As with the previous Gaussian population, 1000 populations of 100 by 100 quadrants were generated for each level of Moran's I, and then 1000 samples were selected from each generated population. The simulated variance of the HT estimator was compared for the populations with different levels of Moran's I in BAS and SRS. Figure 4-5 and Table 4-2 show the results of using two different sampling methods for different populations when $p = 0.5$ and for different sample sizes. The ratio of the simulated variance of the HT estimator of the BAS method to the variance of the HT estimator of the SRS, $r_{BAS/SRS}$, for each sample size is also included in Table 4-2 and the relevant graphs are shown in Figure 4-6.

Table 4-2 Simulated variance of the HT estimator for eight binary populations with different levels of Moran's I when $p = 0.5$ and BAS and SRS are used to select samples of size n=50, 100, 150, 200, 250, 300 and 350.

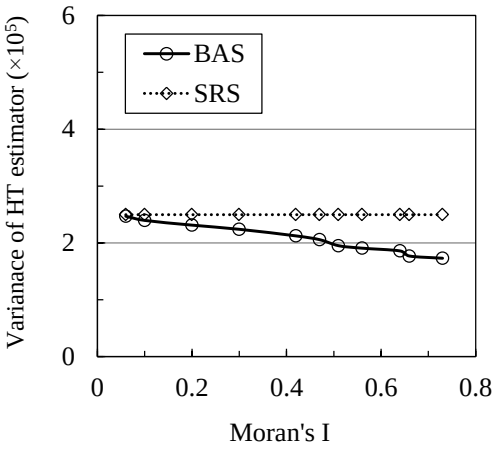
Simulated variance of the HT estimator ($\times 10^5$)								
Moran' s I	Design	Sample size						
		50	100	150	200	250	300	350
0.06	BAS	4.97	2.47	1.62	1.21	0.96	0.79	0.68
	SRS	4.99	2.50	1.66	1.25	1.00	0.83	0.71
	$r_{BAS/SRS}$	1.00	0.99	0.97	0.97	0.96	0.95	0.95
0.10	BAS	4.91	2.40	1.57	1.16	0.92	0.75	0.64
	SRS	4.99	2.50	1.66	1.25	1.00	0.83	0.71
	$r_{BAS/SRS}$	0.98	0.96	0.94	0.93	0.92	0.90	0.89
0.20	BAS	4.78	2.31	1.50	1.10	0.86	0.70	0.59
	SRS	4.99	2.50	1.66	1.25	1.00	0.83	0.71
	$r_{BAS/SRS}$	0.96	0.93	0.90	0.88	0.86	0.84	0.83
0.30	BAS	4.65	2.24	1.45	1.05	0.82	0.66	0.57
	SRS	4.99	2.50	1.66	1.25	1.00	0.83	0.71
	$r_{BAS/SRS}$	0.93	0.90	0.87	0.84	0.82	0.80	0.80
0.42	BAS	4.54	2.12	1.38	1.00	0.78	0.62	0.53
	SRS	4.99	2.50	1.66	1.25	1.00	0.83	0.71
	$r_{BAS/SRS}$	0.91	0.85	0.83	0.80	0.78	0.75	0.74
0.47	BAS	4.38	2.06	1.31	0.96	0.74	0.60	0.50
	SRS	4.99	2.50	1.66	1.25	1.00	0.83	0.71
	$r_{BAS/SRS}$	0.88	0.82	0.79	0.77	0.74	0.72	0.70
0.51	BAS	4.27	1.95	1.25	0.91	0.70	0.57	0.48
	SRS	4.99	2.50	1.66	1.25	1.00	0.83	0.71
	$r_{BAS/SRS}$	0.85	0.78	0.75	0.73	0.70	0.68	0.67
0.56	BAS	4.16	1.91	1.21	0.88	0.68	0.55	0.46
	SRS	4.99	2.50	1.66	1.25	1.00	0.83	0.71
	$r_{BAS/SRS}$	0.83	0.76	0.73	0.71	0.68	0.66	0.64
0.64	BAS	4.04	1.86	1.17	0.85	0.65	0.52	0.44
	SRS	4.99	2.50	1.66	1.25	1.00	0.83	0.71
	$r_{BAS/SRS}$	0.81	0.74	0.70	0.68	0.65	0.63	0.61

Table 4-2 Continued

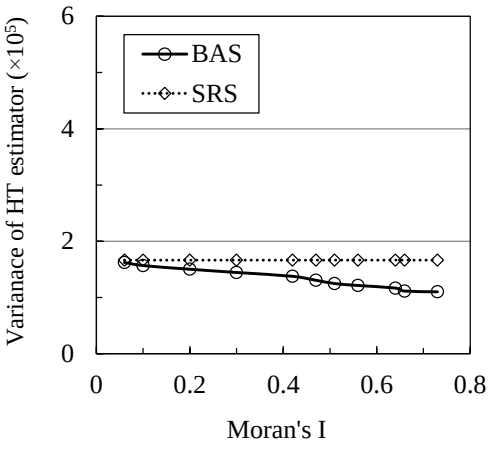
Simulated variance of the HT estimator ($\times 10^5$)								
Moran' s I	Design	Sample size						
		50	100	150	200	250	300	350
0.66	BAS	3.93	1.77	1.11	0.81	0.63	0.50	0.42
	SRS	4.99	2.50	1.66	1.25	1.00	0.83	0.71
	$r_{BAS/SRS}$	0.79	0.71	0.67	0.65	0.63	0.60	0.59
0.73	BAS	3.85	1.73	1.10	0.80	0.61	0.49	0.41
	SRS	4.99	2.50	1.66	1.25	1.00	0.83	0.71
	$r_{BAS/SRS}$	0.77	0.69	0.66	0.64	0.61	0.59	0.57



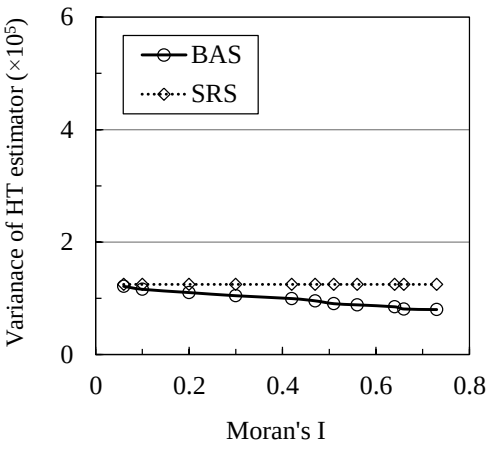
(a)



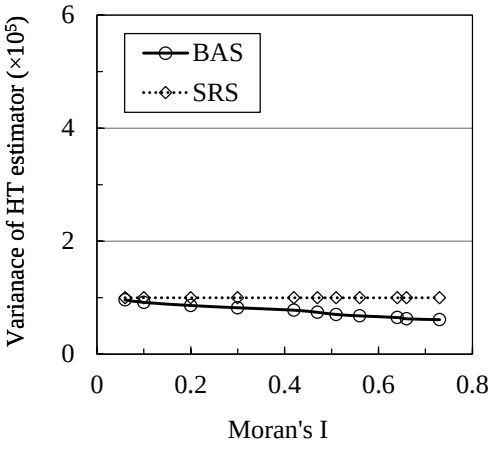
(b)



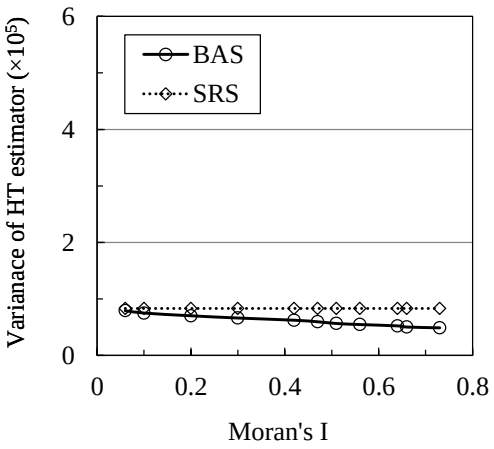
(c)



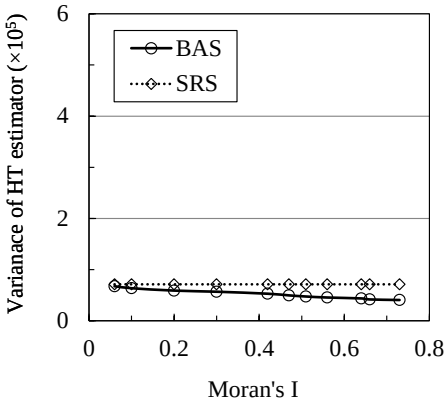
(d)



(e)



(f)



(g)

Figure 4-5 Trend of the estimated variance of the HT estimator for different levels of Moran's I in Bernoulli populations with $p = 0.5$ *when BAS and SRS are used to select different sample sizes (a)* $n = 50$*, (b)* $n = 100$*, (c)* $n = 150$*, (d)* $n = 200$*, (e)* $n = 250$*, (f)* $n = 300$ *and (g)* $n = 350$*.*

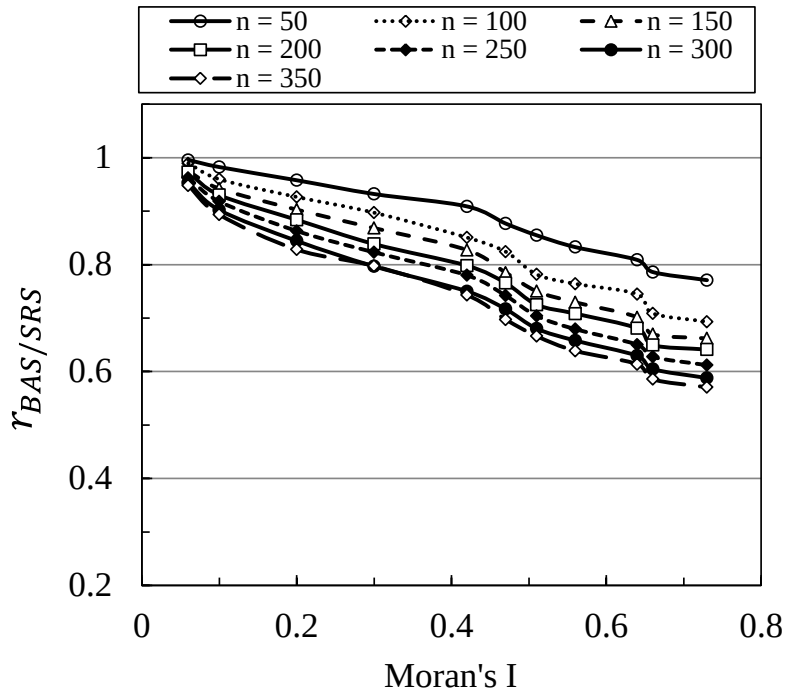


Figure 4-6 The ratio of the variance of the HT estimator of the BAS method to the variance of the HT estimator of the SRS, $r_{BAS/SRS}$, for populations with Bernoulli distribution for different levels of Moran's I.

Results from Figure 4-5, Figure 4-6, and Table 4-2 show that for all sample sizes, there is not a clear trend among the estimated variance of the HT estimator for different levels of Moran's I when samples were selected by SRS. However, by increasing the spatial autocorrelation among population units, the variance of the HT estimator decreased, as expected, most notably with larger sample sizes.

Results showed that the implementation of the BAS method in spatially auto-correlated populations with binary responses can provide more precise estimates than SRS, and this precision will increase as the spatial autocorrelation increases. This ensures that irrespective of the type of variable, by increasing the spatial autocorrelation, the precision of the estimates will increase if the BAS method is used for selecting samples.

4.3 BAS for Stratified Populations

4.3.1 Considering Same Sampling Fraction in Each Stratum

In some situations, the region of the population of interest may be partitioned into strata based on geographical considerations. As described in Chapter 2, stratified sampling is a well-known method that is recommended to deal with this kind of population. Although the application of a stratified sampling method is straightforward, there are a number of aspects that should be considered when applying it. Defining boundaries between strata is one of these aspects that requires time and effort.

One advantage of stratified sampling is that it permits different sampling fractions[1] to be applied in different strata. But, this advantage is less important if disproportionate stratified sampling is not desired (Lynn, 2019). In fact, stratification is sometimes introduced to only ensure that the different sub-regions in the population are represented adequately in the sample. Therefore, a question is raised as to whether stratified sampling could be substituted with a spatially balanced design (such as BAS) whenever we are interested in applying the same sampling fraction in each stratum and there is no interest in providing individual estimates for each stratum.

With BAS in equal probability sampling, sampling units are evenly spread over the area of the population of interest (Robertson et al., 2013). With such even spread there is an expectation that the number of sampling units that would be selected over a specific part of the area will be proportional to the size of that part. This suggests that applying the BAS method without defining boundaries between strata, one can select samples as a stratified sampling method using proportional allocation. Here this will be explored by conducting a simulation study on the population of crabs (which were introduced in Chapter 3).

Suppose that the area of the population of crabs is partitioned into four strata including 58979, 59369, 31809, and 9843 quadrats, respectively. The stratified population of crabs is illustrated in Figure 4-7.

[1] In stratified sampling, the sampling fraction for each stratum is the ratio of the size of the sample to the size of the stratum (Dodge & Marriott, 2003).

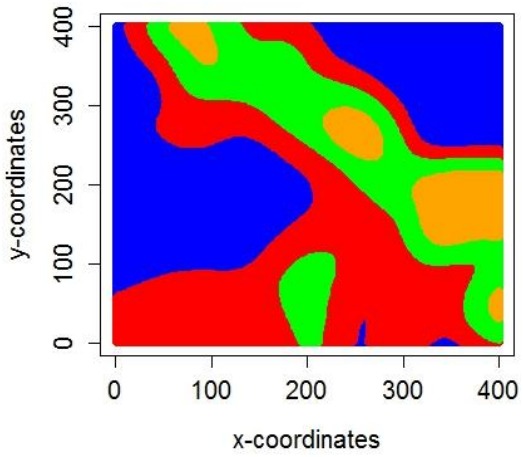


Figure 4-7 Study area of the population of crabs, which is partitioned into four different strata.

In the simulation study, irrespective of explicit strata boundaries, 1000 samples of different sizes ($6^2, 9^2, 11^2, 13^2, 14^2, 16^2$ and 17^2 quadrats) were selected from the population of crabs using SRS and BAS. For each sample, the number of selected quadrats which lay within each stratum was counted. Let m_{hr} ($h = 1, 2, 3, 4$, $r = 1, \dots, 1000$) be the number of quadrats observed in stratum h at the r^{th} iteration. The average and variance of m_{hr} among 1000 iterations for both BAS and SRS are shown in Table 4-3.

If a stratified sampling method with proportional allocation were used, the number of observed quadrats (observed sample sizes) in each stratum would be proportional to the number of quadrats in that stratum. Proportional sample sizes are shown in rows entitled "proportional" in Table 4-3.

Table 4-3 shows that by using either BAS or SRS, the observed average sample sizes within the strata are close to what would be expected if stratified sampling with proportional allocation had been used. However, as can be seen in Table 4-3, the variance of the observed sample sizes in each stratum over the 1000 simulations with BAS is much smaller than with SRS. This means that BAS can produce sample sizes close to what would be observed with stratified sampling and proportional allocation. These results suggest that BAS can be an alternative to sampling methods that select samples from each stratum proportional to the population size of the stratum (i.e., stratified proportional allocation). The merit of using BAS for selecting samples can be

mainly attributed to the fact that it avoids extra effort required for defining boundaries between strata.

It is worth mentioning that ignoring explicit stratifications leads to the loss of ability to obtain estimates in each separate stratum. Therefore, using the BAS method as an alternative to the stratified method with proportional allocation would be suggested only when there is no interest in obtaining information from each stratum. Note that, in the case of ignoring explicit stratifications, post-stratification (stratification after the selection of a sample) techniques (Skinner et al., 1989) can be used to improve the efficiency of estimators.

To understand if there is a change in precision in the estimates when a stratified sampling is substituted with BAS, another simulation study on the population of crabs was performed. For this, 1000 samples of sizes $6^2, 9^2, 11^2, 13^2, 14^2, 16^2$ and 17^2 were selected using BAS within each stratum and BAS without attention to the explicit strata boundaries. The allocated sample size in each stratum was calculated using a proportional allocation method. For each sample, the HT estimator for the total number of crab burrows in the study area was computed. The simulated variance of the achieved HT estimators among 1000 simulated samples ($\widehat{\mathrm{Var}}(\hat{\mathrm{Y}}_{\mathrm{HT}})$) for the two different sampling schemes were calculated using Equation (3.10). In this study, the estimated variance of each sample was also calculated using the local mean variance estimator (Equation (2.18)). For each sample size, the average of the estimated variances among 1000 samples ($\widehat{\mathrm{Var}}(\hat{\mathrm{Y}}_{\mathrm{HT}})_{est}$) was calculated by:

$$\widehat{\mathrm{Var}}(\hat{\mathrm{Y}}_{\mathrm{HT}})_{est} = \frac{1}{1000}\sum_{r=1}^{1000}\hat{\mathrm{V}}_{\mathrm{NBH-r}}(\hat{Y}_T), \tag{4.2}$$

where $\hat{\mathrm{V}}_{\mathrm{NBH-r}}(\hat{Y}_T)$ is the local mean variance that was estimated from the r^{th} sample. Calculated $\widehat{\mathrm{Var}}(\hat{\mathrm{Y}}_{\mathrm{HT}})$ and $\widehat{\mathrm{Var}}(\hat{\mathrm{Y}}_{\mathrm{HT}})_{est}$ are shown in Table 4-4. Figure 4-8 also plots $\widehat{\mathrm{Var}}(\hat{\mathrm{Y}}_{\mathrm{HT}})$ for two different sampling methods.

Table 4-3 Average and variance of the observed quadrats in each stratum for 1000 samples selected by BAS and SRS for a range of different sample sizes. Sample sizes allocated to each stratum if stratified sampling with proportional allocation were applied, are shown in rows entitled "proportional".

Sampling design	Sample size	Stratum 1		Stratum 2		Stratum 3		Stratum 4	
		average	var	average	var	average	var	average	var
proportional		$n_1 = 13$		$n_2 = 13$		$n_3 = 7$		$n_4 = 2$	
BAS	6^2	13.21	1.95	13.48	4.73	7.15	3.38	2.17	1.12
SRS		13.34	8.01	13.4	8.24	7.06	5.67	2.21	1.99
proportional		$n_1 = 30$		$n_2 = 30$		$n_3 = 16$		$n_4 = 5$	
BAS	9^2	29.8	3.26	30.22	6.26	16.01	6.03	4.98	1.36
SRS		29.93	19.4	30.11	18.3	16.12	12.4	4.85	5.1
proportional		$n_1 = 45$		$n_2 = 45$		$n_3 = 24$		$n_4 = 7$	
BAS	11^2	44.67	3.52	44.93	8.43	24.07	7.03	7.33	1.61
SRS		44.26	29.4	45.08	30.1	24.05	19.3	7.61	6.79
proportional		$n_1 = 62$		$n_2 = 63$		$n_3 = 34$		$n_4 = 10$	
BAS	13^2	62.17	4.34	62.77	10.4	33.78	8.64	10.28	2.06
SRS		62.13	40.9	62.88	38.7	33.74	27.7	10.27	9.4
proportional		$n_1 = 72$		$n_2 = 73$		$n_3 = 39$		$n_4 = 12$	
BAS	14^2	72.2	5.03	72.9	9.89	38.92	8.75	11.98	2.38
SRS		72.94	45.4	72.28	46.3	38.69	30.9	12.1	11.6
proportional		$n_1 = 94$		$n_2 = 95$		$n_3 = 51$		$n_4 = 16$	
BAS	16^2	94.22	5.79	95.29	11.1	50.81	9.01	15.68	2.56
SRS		94.34	57	95.16	58	50.73	41.1	15.77	14.7
proportional		$n_1 = 107$		$n_2 = 107$		$n_3 = 57$		$n_4 = 18$	
BAS	17^2	106.47	5.82	107.6	8.97	57.31	9.23	17.63	2.52
SRS		106.12	65.6	107.5	65	57.57	46.9	17.81	15.9

Table 4-4 Simulated variance of the achieved HT estimator for 1000 simulated samples and the average of the estimated variances for 1000 samples selected by two different sampling designs (BAS with proportional allocation and BAS).

Sample size	Sampling design	$\widehat{\mathrm{Var}}(\hat{Y}_{HT})_{SIM}$	$\widehat{\mathrm{Var}}(\hat{\mathrm{Y}}_{\mathrm{HT}})_{est}$
6^2	BAS	76×10^8	81×10^8
	BAS with proportional allocation	28×10^8	29×10^8
9^2	BAS	31×10^8	42×10^8
	BAS with proportional allocation	10×10^8	15×10^8
11^2	BAS	9×10^8	23×10^8
	BAS with proportional allocation	5×10^8	11×10^8
13^2	BAS	5×10^8	12×10^8
	BAS with proportional allocation	4×10^8	7×10^8
14^2	BAS	4×10^8	8×10^8
	BAS with proportional allocation	3×10^8	6×10^8
16^2	BAS	2×10^8	5×10^8
	BAS with proportional allocation	2×10^8	5×10^8
17^2	BAS	1×10^8	4×10^8
	BAS with proportional allocation	1×10^8	4×10^8

Table 4-4 and Figure 4-8 show that for smaller sample sizes, the average of the estimated variance among 1000 samples and the simulated variance of the HT estimator achieved by BAS with proportional allocation is lower than the achieved variance using BAS without considering explicit stratification. However, by increasing the sample size, the two methods provide estimates with almost similar precisions. This suggests that in cases where proportional allocation is desired, BAS with no stratification can be used as an alternative if information from an individual stratum is not required. In fact, stratified sampling with proportional allocation is often used with the primary purpose of ensuring near-even sample intensity over the population. If this is the main reason for using stratified sampling with proportional allocation, employing a non-stratified BAS method can achieve that goal.

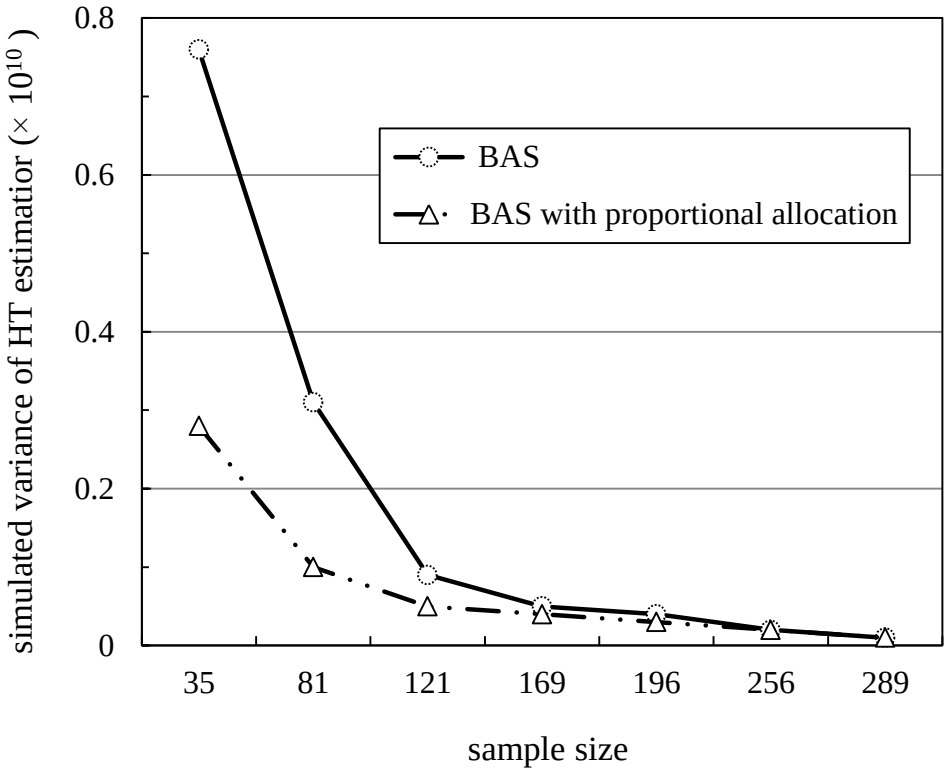


Figure 4-8 Variance of the achieved HT estimator among 1000 simulated samples selected by two different sampling designs (BAS with proportional allocation, and BAS) for a range of sample sizes.

4.3.2 Considering Different Sampling Fractions in Strata

During selection of a sample from a stratified population, it is sometimes desirable to apply different sampling fractions in strata. This could be done by implementing the BAS method independently in each stratum. Another approach for selecting spatial samples from the stratified populations might be the use of a method, called Stratified BAS (StratBAS) in this study. To select sampling units with StratBAS, BAS is simply implemented over the entire area of the population. Sample points that fall within the strata are accepted as sampling units. Once the sample size for a certain stratum is reached, further Halton points that fall within that stratum will be discarded from the sample selection process (Jaksons, 2014).

However the question of whether StratBAS can be used instead of BAS is still largely unstudied. In this section, a simulation study on the population of crabs was

conducted in an attempt to compare the performance of the BAS and StratBAS methods in terms of spreading sampling units over the population.

For this simulation study, 100 samples of sizes $6^2, 9^2, 11^2, 13^2, 14^2, 16^2$ and 17^2 were selected using BAS in each stratum and StratBAS. Here, samples were selected from strata using different sampling fractions. In this simulation study, it is assumed that population units located in each stratum have equal probability of being selected in the sample which is proportional to the size of the stratum.

After calculating the mean of the square of the inclusion probabilities in Voronoi polygons based on Equation (2.22), ζ, for each iteration, the average of the ζ was computed among all replications, $\hat{\mu}(\zeta)$. The results of the simulations are shown in Table 4-5.

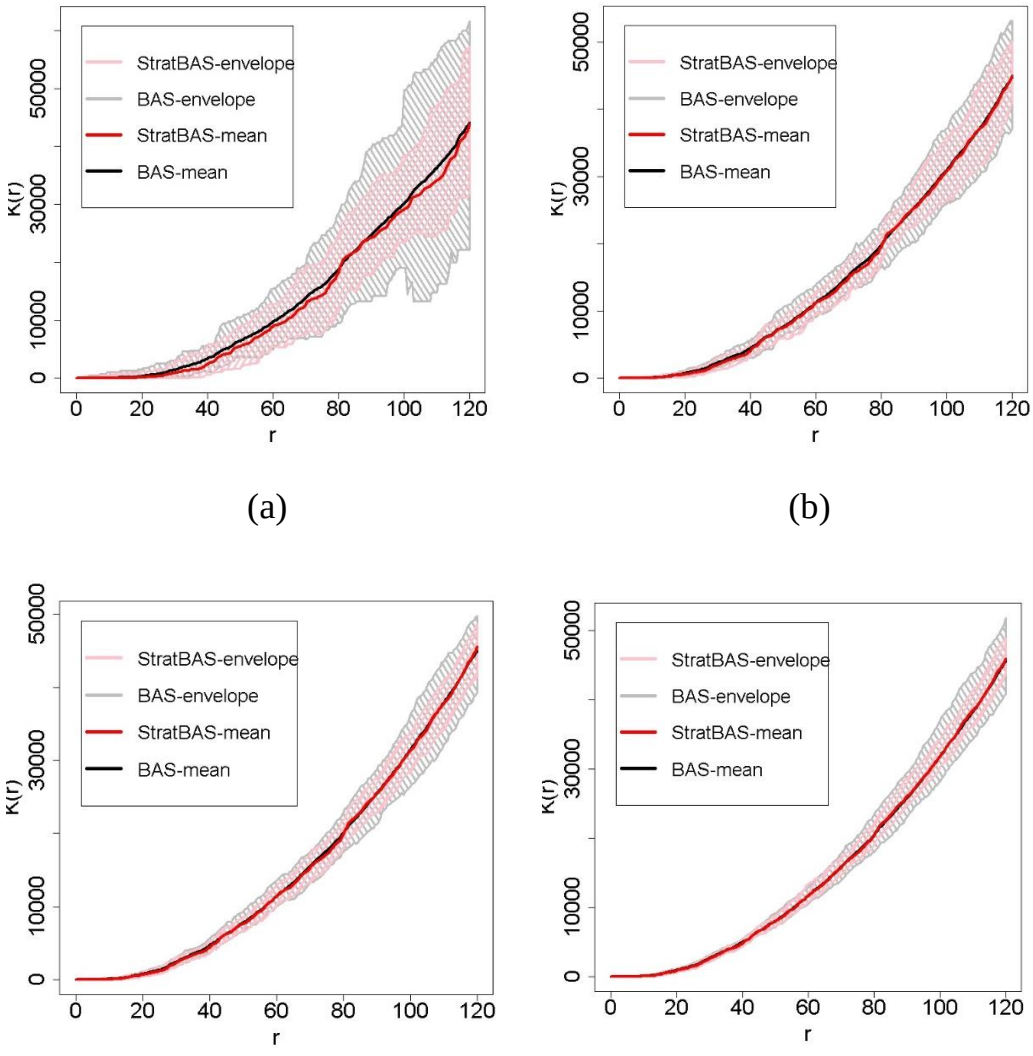
Table 4-5 The average of ζ for two sampling schemes (BAS with stratified sampling and StratBAS) in different sample sizes.

Sample size	$\hat{\mu}(\zeta)$	
	BAS with stratified sampling	StratBAS
6^2	0.20	0.16
9^2	0.21	0.19
11^2	0.22	0.20
13^2	0.22	0.20
14^2	0.22	0.19
16^2	0.23	0.21
17^2	0.23	0.21

Table 4-5 shows that the values of $\hat{\mu}(\zeta)$ related to StratBAS are slightly smaller than the values of $\hat{\mu}(\zeta)$ related to BAS with stratified sampling, for all sample sizes. However, on closer inspection, for all sample sizes, the difference between the values of $\hat{\mu}(\zeta)$ among these two methods is less than 0.04. This shows that there is no remarkable difference between the performance of BAS and StratBAS in spreading sampling units over the population. As mentioned before, the exact values of the inclusion probabilities of the population units in the StratBAS method could not be calculated here. So results showed in Table 4-5 may not be satisfactorily reliable. To make sure that both BAS and StratBAS work similarly to each other in terms of

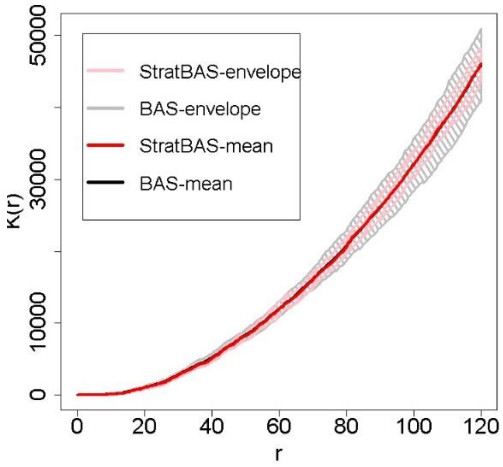
spreading samples over the population, the Ripley's *K* function, $\widehat{K}(r)$, was also used as another tool to compare the spatial balance of these two sampling techniques.

To apply Ripley's *K* function, $\widehat{K}(r)$ for all samples selected by BAS with stratified sampling, and StratBAS, was calculated for a range of radii (r). The study area of the population of crabs covers a 400 × 400 square meter, therefore $\widehat{K}(r)$ here is calculated for distances up to 120 m in increments of 1 m. Figure 4-9 shows the polygons (envelopes) of the boundaries of the calculated $\widehat{K}(r)$. The averages of the calculated $\widehat{K}(r)$ are also plotted in Figure 4-9. Note that, for distances more than 120 m, the differences between the upper and lower bound of the envelopes decreases such that envelopes shrink to a line. Because of this the results are reported up to 120 m.

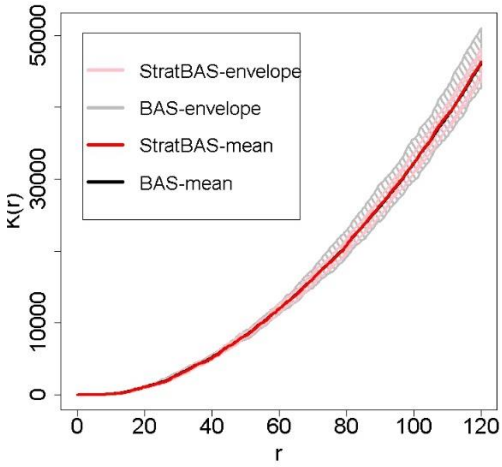


(a) (b)

(c) (d)



(e)



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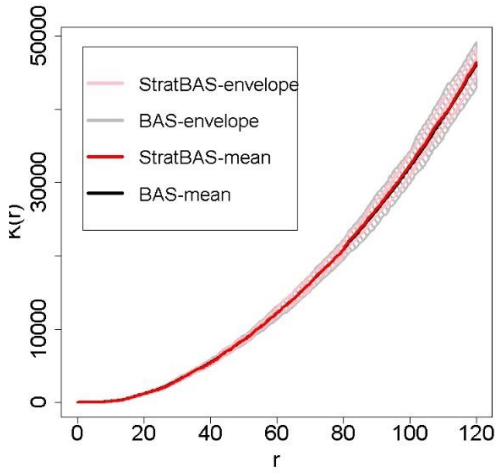

(g)

Figure 4-9 Polygons (envelopes) of boundaries of samples selected by BAS with stratified sampling and StratBAS along with average values of the calculated $\widehat{K}(r)$ for a range of sample sizes (a) $n = 36$, (b) $n = 81$, (c) $n = 121$, (d) $n = 169$, (e) $n = 196$, (f) $n = 256$, (g) $n = 289$.

As can be seen from Figure 4-9, for all radiuses, the Ripley's *K* functions obtained for all samples selected by BAS and StratBAS are close to each other. This confirms that there is no difference between these two methods in spreading the sampling units over the population.

Even though results achieved from the simulation study showed that StratBAS is able to control the spatial balance over the whole population, there are some issues with this technique. In the StratBAS method, BAS is not carried out separately within each

stratum, causing the sampling selection process in each stratum to be dependent on other strata. Hence, the inclusion probabilities of the population units are difficult to compute. In addition, the formulas used in the common stratified sampling method may not be applicable for estimating the parameters of interest with StratBAS. This may lead to an estimation of the parameters with a bigger variance.

Hence, considering the advantages of the BAS method where it is applied within each stratum, over the StratBAS method, the use of BAS within each stratum independently is recommended instead of StratBAS.

4.4 Conclusions

This chapter evaluated the effect of spatial autocorrelation of observations on the precision of the population estimates when BAS is used to select the sample. To test this, two simulation studies were conducted on two different types of populations (populations where responses have Gaussian and Bernoulli distribution). The results of the simulation studies showed that in both populations, by increasing the spatial autocorrelation (which is measured by Moran's I in this thesis), the precision of the population estimates increased compared to selecting the sample by SRS.

This chapter also investigated the application of BAS in stratified populations. For this, simulation studies were performed on a population derived from a study of counts of crab burrows. The simulation studies showed that BAS can be considered as an alternative method for stratified sampling when proportional allocation is designed. In fact, when BAS is used as an alternative to proportional allocation in stratified sampling, it has the advantage that there is no need to create explicit strata.

In situations that need to use different sampling fractions in strata, samples can be selected either by applying BAS in each stratum independently or implementing a technique called StratBAS. This chapter through conducting a simulation study showed that BAS and StratBAS methods have similar performances in terms of spreading sampling units over the population.

In this chapter it was found that the BAS method has a number of advantages when it is used in practical settings in environmental studies. This indicates that the BAS method has promising potential to be extended to other surveys (i.e., household

sampling surveys). In the following chapters, I will investigate this matter by trying to apply the BAS method and other spatially balanced sampling methods in selecting sampling units in social studies.

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Chapter 5 *Spatially Balanced Sampling Methods for Household Surveys*

5.1 Introduction

Household surveying is inherently a spatial science because it almost always involves human populations living in specific geographic regions. Viewed in this way, the human and physical characteristics of locations where people live and the interrelationships among neighbouring people should be considered when designing a sampling survey. Several studies in the literature have reported that people with similar socio-economic characteristics (for example, income, occupation, education) tend to live close to each other. For instance, a study conducted by Kalogirou and Hatzichristos (2007) on modelling the income estimation in Athens in 2001 showed that there was a very strong relationship between the mean household income in each area and the proportion of its residents with high levels of education. In addition, the study indicated that relationship between income and education was not stationary across the areas of Athens, and the data were spatially autocorrelated. In another study, Cuberes and Roberts (2015) found that there was a positive correlation between a household's income and the distance from their home to the city centre in Britain. In two separate studies, Kantar and Aktaş (2016) and Alves (2012) also investigated the pattern of unemployment rate over the provinces in Turkey and subsections of Porto city in Portugal, respectively. The results of both studies showed that there was a spatial dependency between neighbouring regions in terms of unemployment rate.

The fact that there is often a spatial similarity between neighbouring people, households or regions means that selecting nearby sampling units (i.e., people, households, regions) can provide us with similar information. In household surveys, these samples (that contain nearby units) are considered as undesirable samples and should be avoided as much as possible in sample selection.

In the previous chapters, spatially balanced sampling methods were introduced as sampling methods that avoid selecting nearby units. Although these methods have been widely used in environmental surveys, they have not been applied in sampling of human

populations and household surveys. This chapter considers the application of spatially balanced sampling methods in household surveys. Firstly, some general features of household surveys will be discussed, and their dissimilarities with environmental studies will be explained in the second section. The third section will introduce a sampling frame for applying the balanced acceptance sampling (BAS) method more efficiently with discrete populations. Statistical and spatial properties of BAS with this frame will be compared with other available spatially balanced sampling methods. The fourth section will study the application of the spatially balanced sampling methods on stratified populations.

5.2 Spatially Balanced Sampling Methods in Environmental Studies Versus Household Surveys

Spatially balanced sampling methods have been designed initially for studies of natural resources and environmental phenomena, such as air, water, soil, etc. Before applying the spatially balanced sampling methods in household surveys, it is helpful to consider how their objectives and target populations differ from those in environmental studies.

The main objective of household surveys is to provide a comprehensive range of data relating to the socio-economic aspects of people and households across different geographical regions, and ethnic or cultural groups. In household surveys, usually, the required information is gathered from households or individuals by visiting their places of residence (e.g., dwelling or housing units). Since the number of sampling units in household surveys is finite (although it might be unknown), they constitute a discrete population. In this case, the sampling units can be spatially distinguished from each other.

Environmental studies, on the other hand, usually are aimed to assess the status and condition of natural resources. Studies of natural resources and environmental phenomena often need to gather information from linear or areal populations that comprise an infinite number of points in one or more dimensions (Gregoire & Valentine, 2007). In fact, characterizing the status of natural resources in a particular region, called the study area, is a common objective in environmental surveys and this characterization can be met by visiting spatial locations over the region, i.e., point sampling of a continuous population (Pfeffermann & Rao, 2009). The entire surface of a lake and the length of a river are two examples of continuous populations in environmental surveys.

Figure 5-1 provides Google images of (a) a discrete population that contains a set of housing units in a small region of a city, and (b) a continuous population which is an agricultural field. The target population in Figure 5-1a consists of people who are living in the housing units in this region, while the target population in Figure 5-1b is the entire surface of the agricultural field.

(a) (b)

Figure 5-1 Google images of (a) a discrete population (www.hnzc.co.nz) and (b) a continuous population (www.financialtribune.com).

A spatial sampling design on a continuous environmental population may be conducted by selecting some geographical locations at regular distances from the map of the study area, and then observing the attributes of interest corresponding to the selected locations. In a simple format, this can be done by overlaying a regular grid of cells[1] over the map of the study area and then considering the centre of these grid cells as sampling units. By this, the continuous population is converted to a discrete population. Other alternative methods for defining the sampling units in the grid cells method can be found in Olea (1984).

Alternatively, one can generate some random points over the study area and then consider a district area around each point as sampling areas. An example of a spatial sample that is selected from a continuous population is shown in Figure 5-2.

[1] In spatial sampling, grid cells are defined in user-definable size, shape (triangular, hexagonal, square, linear strips or random rectangular), and orientation.

Figure 5-2 A spatial sample selected from a continuous population.

However, these methods could not easily be applied on finite discrete populations that consist of housing units, especially if the spatial pattern of the population tends to be clumped rather than uniform. In this case, generating random points on the map of the region of interest may lead to selection of some areas that have no sampling units (e.g., housing units or dwellings) or some areas that include more than one sampling unit. This limitation is illustrated in Figure 5-3. In order to select a spatial sample from this discrete population, after imposing a grid of cells over the map of the region of interest, some areas are selected as spatial sampling areas. These areas are shown by ✶. Figure 5-3 shows that some of the selected areas do not include any housing units.

Figure 5-3 Sampling areas selected by overlaying a grid on a small part of a city. Selected areas are shown by ✶.

A pragmatic approach that has been used for dispersing the sampling units in a discrete population is to create a linear order of the units that are located in the space of the population

and then use the systematic sampling along the ordered population (Kish, 1965; Geuder, 1984; Pfeffermann & Rao, 2009). This method is popular for spreading the sampling units in the first stage of a multistage cluster sampling. For example, O'Campo et al. (2015) used a serpentine ordering, north to south and east to west for selecting enumeration areas, in order to provide an even spread of the neighbourhoods over the City of Toronto's geography in a study of neighbourhood effect on health and well-being in the City. However, the serpentine ordering does not necessarily avoid selecting neighbouring units in the sample.

In the following sections, the application of the recently developed spatially balanced sampling methods – which were introduced in the previous chapters – in household surveys will be discussed.

5.2.1 Suitability of Balanced Acceptance Sampling for Selecting Samples From Discrete Populations

In the previous chapters, the balanced acceptance sampling **(**BAS) was used for selecting samples from continuous populations. In this section, its application for selecting samples from discrete populations is investigated.

In the context of using the BAS method in a discrete population, a spatially balanced sample can be achieved by handling the partitioning process to divide the population into some equally sized cells in such a way that each cell contains equal numbers of population units. Some algorithms for providing equitable spatial partitions in irregular populations can be found in Bast and Hert (2000) and Carlsson et al. (2010). In another technique, the population units might be surrounded with non-overlapping equal-sized boxes (Robertson et al., 2013). This is done by replacing each point corresponding to each population unit with a box. For implementing the BAS method in this situation, after generating the random start Halton sequence, if the Halton point is located within a unit's box, that unit is selected in the sample. Halton points located outside the boxes will be rejected. An example of a discrete population is shown in Figure 5-4a. For applying the BAS method in this population, equal-sized boxes, as shown in Figure 5-4b, are firstly overlaid around units. Next, some of the boxes are selected as sampling units using the location of Halton points. In this example, the units surrounded by red boxes are selected as sampling units since the Halton points that were generated have been located within these boxes (Figure 5-4c). The rejected Halton points are shown by solid black triangles (Figure 5-4c).

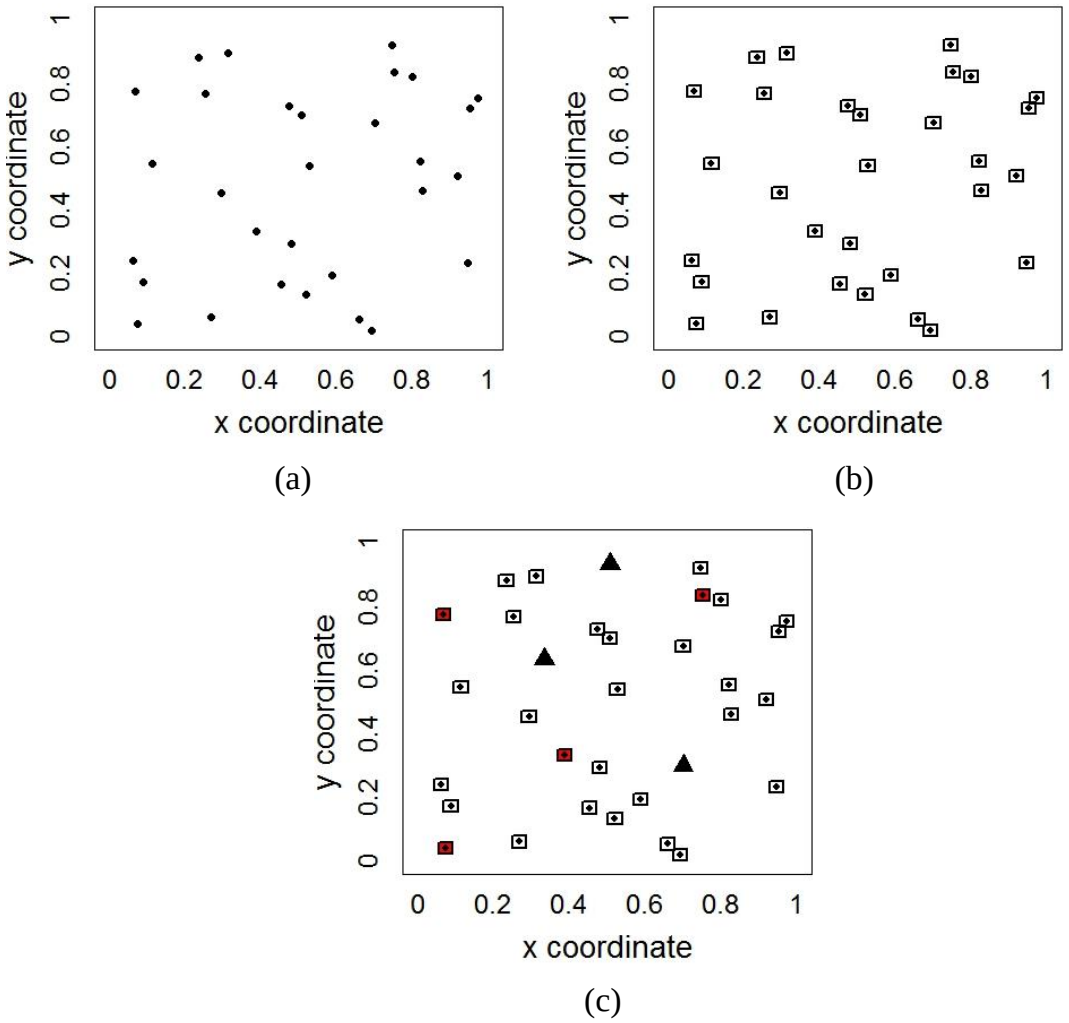


Figure 5-4 (a) An example of a discrete population (b) equal boxes are placed around discrete units, (c) using the BAS method, a unit is selected if the Halton point is within the unit's box. The boxes of four selected sampling units are shown in red. Solid triangles show Halton points are located outside the boxes.

In this technique, the area that defines the acceptance region of the population units is essentially shrunk to the area of boxes. In this situation, an acceptance/rejection sampling can be used to select samples. However, defining equal-sized, non-overlapping boxes around each population unit may be inefficient when the population units are clustered. In fact, in this situation, the area of the boxes is so small that a considerable number of generated Halton points would be rejected. Figure 5-5 shows a discrete population in which sampling units in some parts of the study area are clustered. In this case, considering equal-sized, non-overlapping boxes around units shown within the circles would not be helpful in implementing BAS. Robertson et al. (2017) generated a clustered population of size 1000

units and showed that selecting a sample of size 20 units from this population by BAS, requires approximately 2.5 million random start Halton points. Increasing the number of rejected Halton points can result in sampling units that are not spread evenly over the population. This happens because so many Halton points are skipped during sample selection which may lead to the selection of nearby units. One solution to this situation will be discussed in Section 5-3.

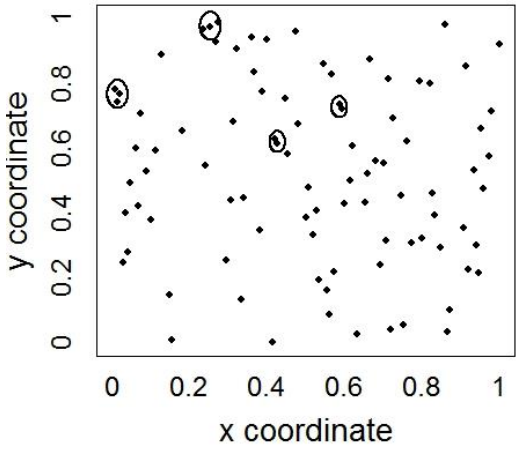


Figure 5-5 An example of a discrete population in which sampling units are located very close to each other. Very close units are shown in circles. Non-overlapping boxes around these units are so small that using BAS would be inefficient.

5.3 A Frame for BAS for Discrete Populations

In this section, a sampling frame will be introduced that makes the application of the BAS method on discrete populations more efficient. The general idea of this technique is to create a spatial frame of the population units and then implement the BAS method to select spatially balanced samples from the created frame. Since the main effort of this technique is mostly to create a suitable spatial sampling frame, it is called the BAS-Frame technique in this thesis. This technique divides the region of the population of interest into some partitions (cells) hierarchically. Then, the BAS method is used to select sample boxes. Employing the BAS method ensures that spatially adjacent cells seldom appear together in the sample. The BAS-Frame technique can be implemented through the following steps:

Step 1- Constructing a Primary Frame

Partitioning the region of the population of interest creates a collection of boxes such that these boxes cover the entire region of the population of interest without any overlap.

These boxes form a primary frame. Creating a primary frame from a discrete population is followed up by successive divisions of population units in the given dimensions (e.g., vertical and horizontal divisions in two dimensional populations). For the vertical division, the region of the population of interest is split along the first coordinate axis so that the number of units in each of the new sub-areas is the same. If the number of population units is odd, an extra unit is either added to or removed from the population randomly and then the units are divided into two parts. Each of these options (adding to or removing random points from the population) has its own advantages and drawbacks which will be studied later in this chapter and the next chapter. Since the partitioning process is based on the density of the units, the boxes can be of different sizes.

Figure 5-6 shows the first vertical division on a discrete population. Data shown in the figure is known as the "Boston Housing Dataset", which was collected by the U.S Census Service on housing in the area of Boston and was originally published by Harrison Jr and Rubinfeld (1978). The population units are divided into two boxes (B1 and B2) along the first coordinate axis (longitude) as shown in Figure 5-6.

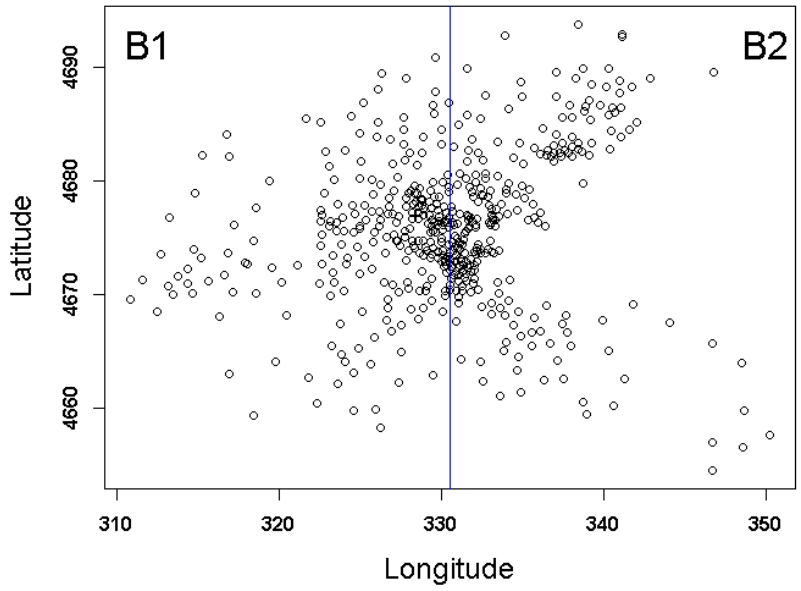


Figure 5-6 The geographical locations of 506 cases in Boston Housing Dataset. The study area is divided vertically into two boxes (B1 and B2). Since the number of units (houses) is even (506), it is not necessary to add an extra unit randomly to it.

In the horizontal division, the units in each created box are divided into two parts with the same count of units based on the second coordinate axis. Since the number of units in

each created box (i.e. 253 units) is odd, before partitioning an extra unit was added to each region randomly (they are shown by red color circles). As mentioned earlier, instead of adding an extra unit randomly, a unit can be removed randomly from the population. Figure 5-7 shows the created boxes after completing the horizontal division on the current case study. The generated boxes are addressed sequentially. For instance after two stages of partitioning process, the generated boxes are designated the labels B13, B14, B23 and B24.

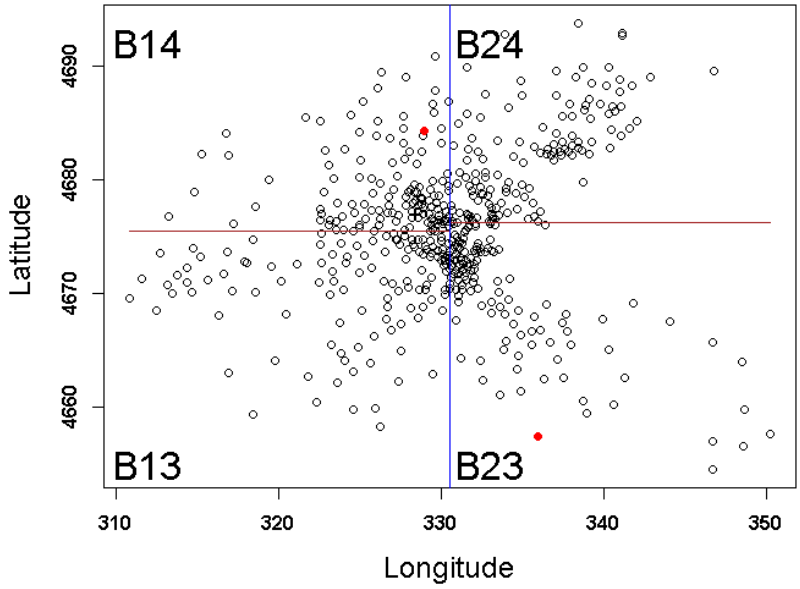


Figure 5-7 Boxes created after the horizontal division. Horizontal division is done in each box achieved in the previous step. In this example, each created box in the first step contains 253 units, so an extra unit (red points) was added randomly to each box. The current boxes are halved with the same count of units.

The process of vertical and horizontal division is continued hierarchically until each box contains only one unit. For example, after 6 partitions, the case study area is split into $2^6 = 64$ boxes (see Figure 5-8). Although, the area of the boxes is different, the number of units in each box is the same. The randomly added units during the splitting process are shown in red in Figure 5-8. For large populations that need many artificial points to be added to them, the process of division can be stopped earlier so that each box contains more than one population unit. This approach for selecting samples will be discussed later.

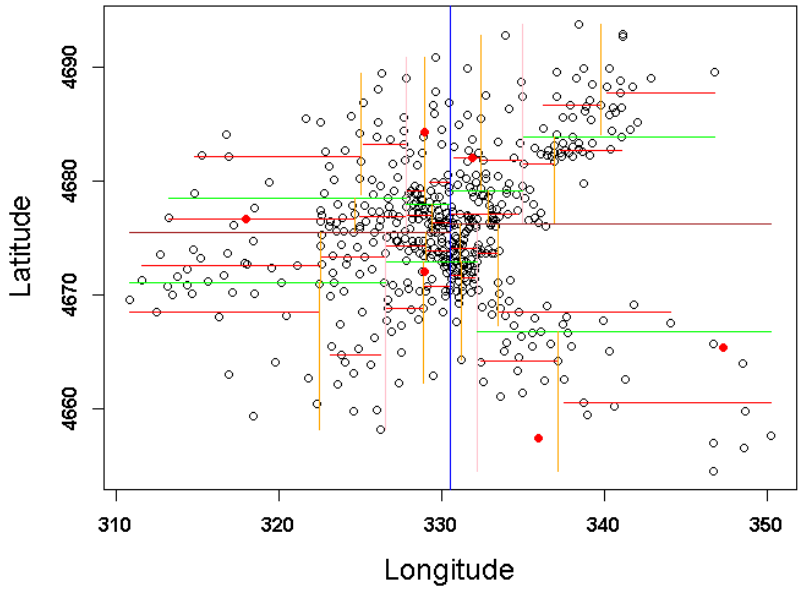


Figure 5-8 Boston Housing data study area split into 64 boxes after the first six levels of the partitioning process. During the partitioning process of the Boston Housing data into 64 boxes with the same counts of units, some units are added randomly; these units are shown in red.

These added points are virtual units which are added only for partitioning the population of interest, so they are assigned zero inclusion probability in the sampling process.

Discrete spatial populations sometimes contain units with identical coordinates. In the Boston Housing Dataset, for example, there are two units that have the same longitude (= 318.54) but with different latitude. These units are shown in red on Figure 5-9. Also, the green units in Figure 5-9 have the same latitude (= 4667.33) and different longitude. The presence of such units in the population might cause problems with the partitioning process.

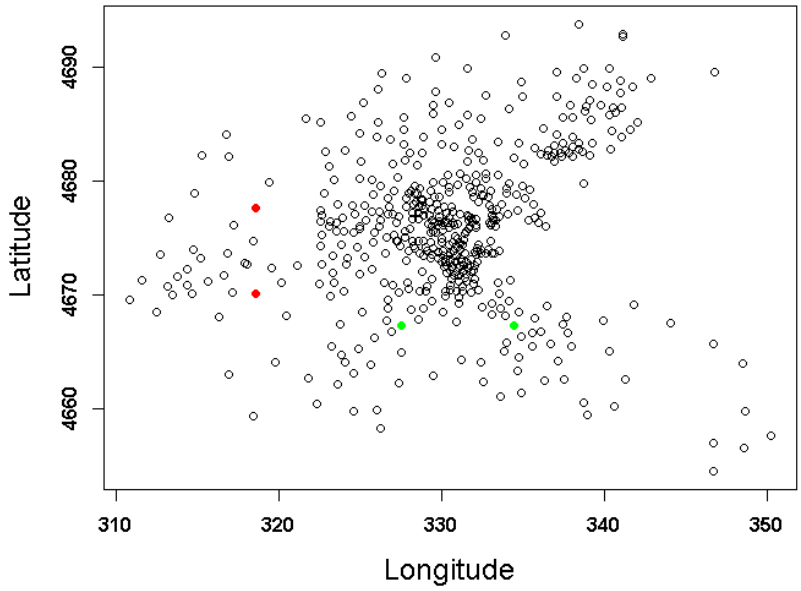


Figure 5-9 Units in the Boston Housing dataset that have the same longitude are shown in red. Green points show units that have the same latitude.

In this study, a jittering technique is used to remove these overlapping units. Jittering is a perturbation technique which adds a random number to every coordinate. The random number is usually simulated from a uniform distribution over an interval or a Gaussian distribution with mean zero and standard deviation σ. In this thesis, $\sigma = d/5$ was considered, where d is the smallest difference between the coordinates. The technique is commonly used to preserve individual privacy (Agrawal & Srikant, 2000) as well as to get rid of units with identical coordinates.

Step 2- Constructing a Regular Frame

In the primary frame shown in Figure 5-8, each box is assigned a unique address based on the order in which the divisions were carried out. These addresses can be placed into a regular frame as shown in Figure 5-10.

In contrast to the primary frame, the boxes in the regular frame have identical area. Therefore, they have the same chance of being selected in the sample when the BAS method is implemented. Note that the boxes corresponding to the added points have zero inclusion probability.

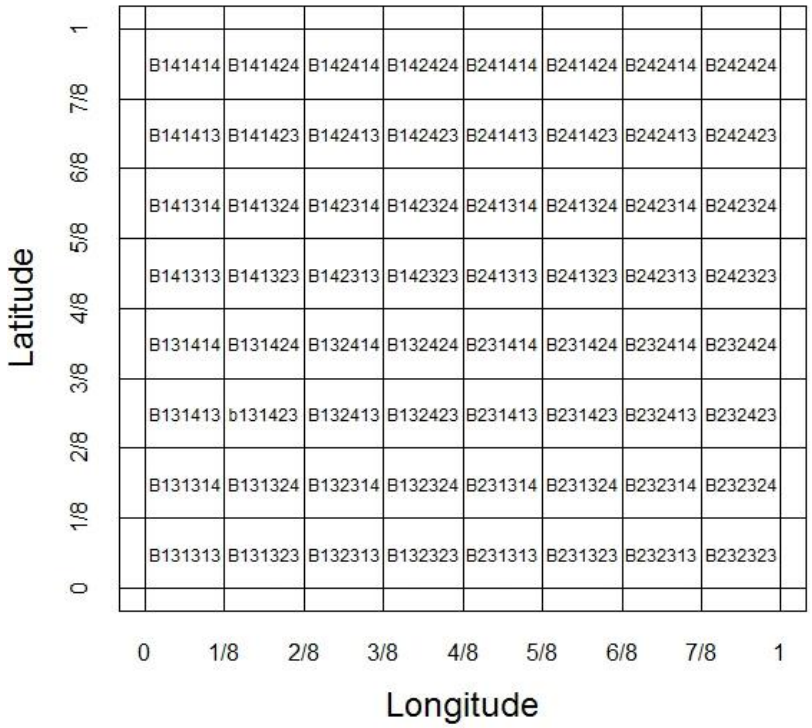
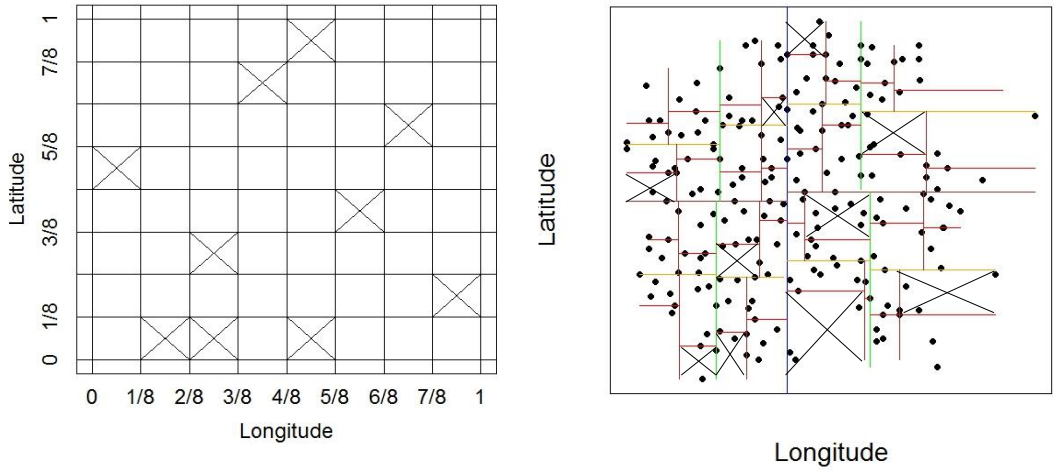


Figure 5-10 A regular frame based on the primary frame shown in Figure 5-8 for selecting equal probability sampling units using the BAS method. This frame contains 64 equal-sized boxes that are addressed the same way as the primary frame.

Step 3- Sampling Unit Selection

After constructing the regular frame, the BAS method can be used to select a sample of n distinct boxes. A box is selected in the sample if the generated Halton point is located within the box's boundary defined in the regular frame. The process of sample selection is continued until n distinct boxes are recorded.

Because there is a one-to-one correspondence between the addressed boxes in the primary frame and those in the regular frame, the units selected on the latter can be mapped back onto the former as shown in Figure 5-11.



(a) (b)

Figure 5-11 (a) Selected boxes using the BAS method from a regular frame, and (b) the location of the selected boxes on the relevant primary frame.

5.3.1 Spatial Properties of the BAS-Frame Technique

To evaluate the spatial balance of the BAS-Frame technique and to compare it with other spatially balanced sampling methods, a simulation study was conducted. Through the simulation study, the spatial balance of five sampling designs (SRS, BAS-Frame, LPM1, GRTS and SCPS) was compared on an artificial finite population that consists of 1024 discrete units with irregular positions. The sample size was chosen such that there was no need to either add or remove random points in the population. This condition represents an ideal situation because the results are not affected by the addition or removal of random points. This population, which is based on an example in Stevens, D. and Olsen (2004), is shown in Figure 5-12. As seen, the population has a high spatial variability; some regions are empty of units, whereas some regions are densely populated.

This simulation study investigated the spatial balance of the evaluated designs by using the quadrat-based method, which is a class of descriptive statistics in spatial point pattern analysis (see Section 2.6.3). This method is based on counts of sampling units that are located within the cells of a regular grid that covers the region of the population of interest. In order to use this method here, the population was divided into 10 × 10 equal square cells. The number of population units in non-empty cells ranged from 1 to 54.

After selecting a sample with a sampling fraction equal to 5% for each sampling design, the number of sampling units that fell into each square cell (achieved sample size for each

square cell) was counted. The sample selection process (including creating frames) was repeated 1000 times, and then the variance of the achieved sample size for each square cell among 1000 repetitions was calculated. Note that the considered designs are all unbiased sampling methods.

In Figure 5-13, the variance of the achieved sample size for each square cell is plotted against the frequency of population units of each square cell.

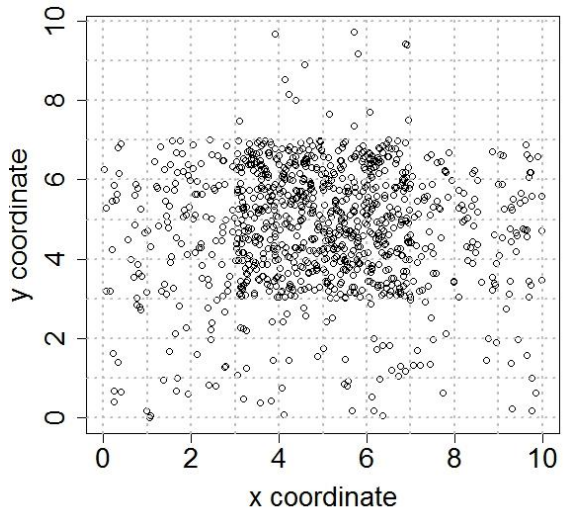


Figure 5-12 An artificial population used in a spatial balance investigation of the BAS-Frame technique, overlaid with a 10 × 10 grid of square cells.

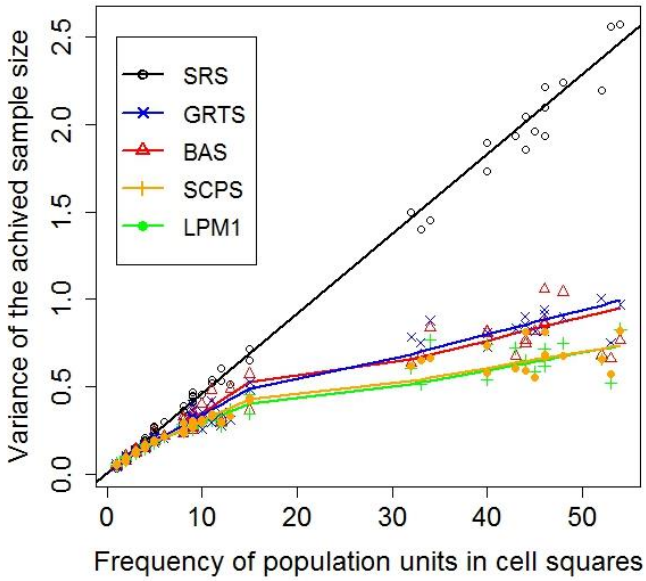


Figure 5-13 Comparison of spatial balance of SRS, GRTS, BAS, LPM1 and SCPS using the quadrat-based method. Results are based on using 1000 samples of size 50. The achieved sample size is the number of samples that fell into each of the 100 square cells.

Figure 5-13 shows that, of all the sampling methods, SRS, as expected, had the largest variance of the achieved sample sizes for all square cells with different numbers of population units. The spatially balanced sampling designs had approximately the same variance. LPM1 had the smallest variance of the achieved sample sizes for all square cells.

In addition to the quadrat-based method, the Voronoi polygons, explained in Equation (2.22), were used to compare the spatial balance between the evaluated designs. Let $\hat{\mu}(\zeta)$, as before, be the average of ζ ($\zeta = \frac{1}{n}\sum_{i\in s}(v_i - 1)^2$ where v_i indicates the sum of the inclusion probabilities of units in the Voronoi polygon related to the i^{th} sampling unit) among all 1000 replications.

where ζ_r is the ζ of the r^{th} iteration. Small $\hat{\mu}(\zeta)$ indicates good spatial balance. The achieved values of $\hat{\mu}(\zeta)$ for a range of selected sample sizes (10, 20, 30, 40 and 50) and for the considered sampling designs are shown in Table 5-1.

Table 5-1 Comparison of the spatial balance of SRS, GRTS, BAS, LPM1 and SCPS using the Voronoi polygons method. The values of $\hat{\mu}(\zeta)$ were estimated from 1000 simulated samples and for five different sample sizes.

Sampling design	Sample size				
	10	20	30	40	50
SRS	0.36	0.35	0.34	0.35	0.35
LPM1	0.15	0.11	0.10	0.10	0.10
BAS	0.15	0.12	0.12	0.12	0.12
GRTS	0.19	0.16	0.14	0.13	0.13
SCPS	0.15	0.11	0.11	0.10	0.10

As seen in Table 5-1, for each selected sample size, SRS, as expected, has the largest values of $\hat{\mu}(\zeta)$ and shows the worst spatial balance among the designs. Of all spatially balanced sampling methods, GRTS has the largest value of $\hat{\mu}(\zeta)$. Again the $\hat{\mu}(\zeta)$ related to LPM1 is better than other spatially balanced sampling methods.

Figure 5-13 and Table 5-1 confirm that the BAS-Frame technique is comparable with other spatially balanced sampling methods in terms of spreading the sampling units over the population.

5.3.2 Statistical Properties of the BAS-Frame Technique

In order to describe the statistical properties of the BAS-Frame technique and compare it with other sampling methods, a simulation study was conducted using the Christchurch Census 2013 meshblocks. A meshblock is the smallest geographic area that constitutes a first-stage sampling frame for most household sampling surveys by Statistics New Zealand (Stats NZ, 2013b). Sample meshblocks are typically selected at the first stage of a household sampling survey by using unequal probability sampling methods. However, the simulation study in this subsection supposes that the meshblocks are the ultimate population units, which should be selected by equal probability sampling techniques.

A map of Christchurch meshblock boundaries and their centres are shown in Figure 5-14.

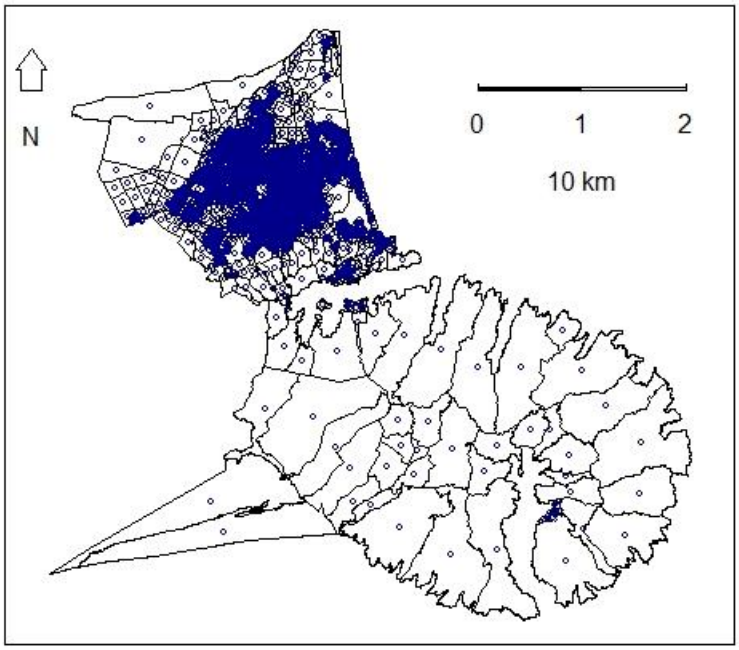


Figure 5-14 A map of Christchurch meshblock boundaries including the centre of each meshblock.

It appears from Figure 5-14 that the meshblocks in Christchurch city vary in size with the smaller ones being situated in the city center and the larger ones in the suburbs.

In order to investigate the efficiency of the BAS-Frame technique for selecting spatially balanced samples from populations with different levels of density, three different levels were considered in Christchurch city. The sampling methods were then applied to each layer separately. The first layer consisted of meshblocks associated with Christchurch city centre.

The second layer covered a larger portion of meshblocks of Christchurch including the first layer as well as suburban areas. The third layer expanded to accommodate the first two layers. These three layers are shown in Figure 5-15. In this study, the density of each layer is defined by dividing the total number of meshblocks located in that layer by their area:

$$Density\ of\ layer\ i = \frac{total\ number\ of\ meshblocks\ which\ are\ located\ in\ layer\ i}{area\ of\ layer\ i},$$

$$for\ i \in \{1,2,3\}. \tag{5.1}$$

Some descriptive information about the three different layers and the whole of Christchurch city is provided in Table 5-2.

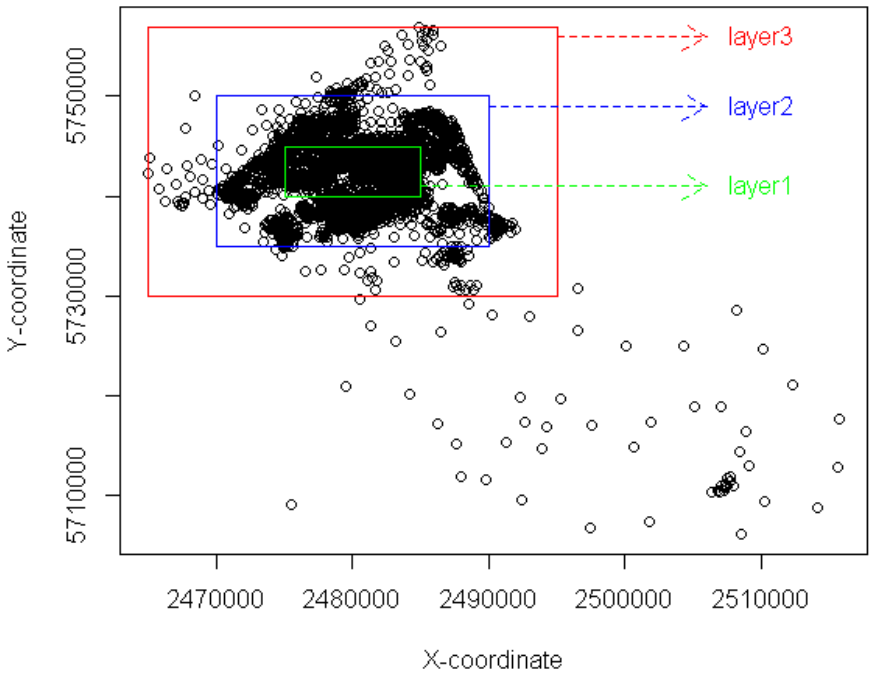


Figure 5-15 Three different layers of Christchurch meshblocks showing their densities.

Table 5-2 Average area of meshblocks (km^2), density, and standard deviation (km^2) of the area of Christchurch city meshblocks (km^2) in each layer.

Layer	Average area of meshblocks	Density	Standard deviation of area of meshblocks
Layer 1	49,07 9	20×10^{-6}	62,745
Layer 2	90,794	11×10^{-6}	285,555
Layer 3	154,246	6.5×10^{-6}	971.021
All Christchurch	511,738	2×10^{-6}	3,651,726

Table 5-2 shows that from layer 1 to layer 3, the density decreases and the standard deviation of meshblock areas increases.

From each layer, samples with five different sampling fractions (1, 2, 3, 4 and 5% of meshblocks) were selected, employing LPM1, BAS-Frame and SRS. For constructing frames for the BAS-Frame method, two scenarios were considered: adding and removing random points. For each sampling design, the sample selection was repeated 1000 times. In the case of the BAS-Frame method, the creation of frames was also repeated 1000 times.

The simulation study investigated the statistical efficiency of the BAS-Frame in a situation when values of the considered response variable have a spatial trend. In this situation a response variable, y, which has a spatial trend, was created using the function

$$y_i = (3(x_{1i} + x_{2i}) + \sin(6(x_{1i} + x_{2i}))) \tag{5.2}$$

where y_i is the response for the i^{th} meshblock, x_{1i} and x_{2i} are longitude and latitude of the centre of the i^{th} meshblock. This function is taken from Grafström et al. (2012).

The spatial trends of the response variable y across three different layers of meshblocks and all Christchurch meshblocks are depicted in Figure 5-16. The number of meshblocks and relevant Moran's I values are also presented in Table 5-3.

Table 5-3 The number of meshblocks and Moran's I value of the response variables y for different layers of Christchurch city meshblocks.

	Layer 1	Layer 2	Layer 3	All of Christchurch
Number of meshblocks	860	2441	2629	2684
Moran's I value of response variable y	0.44	0.40	0.37	0.36

As Figure 5-16 and Moran's I values related to the response variable y (Table 5-3) show, there is a positive spatial trend among response variable y in all layers of Christchurch meshblocks. Therefore, the spatially balanced sampling methods are expected to select more representative samples than non-spatial sampling methods.

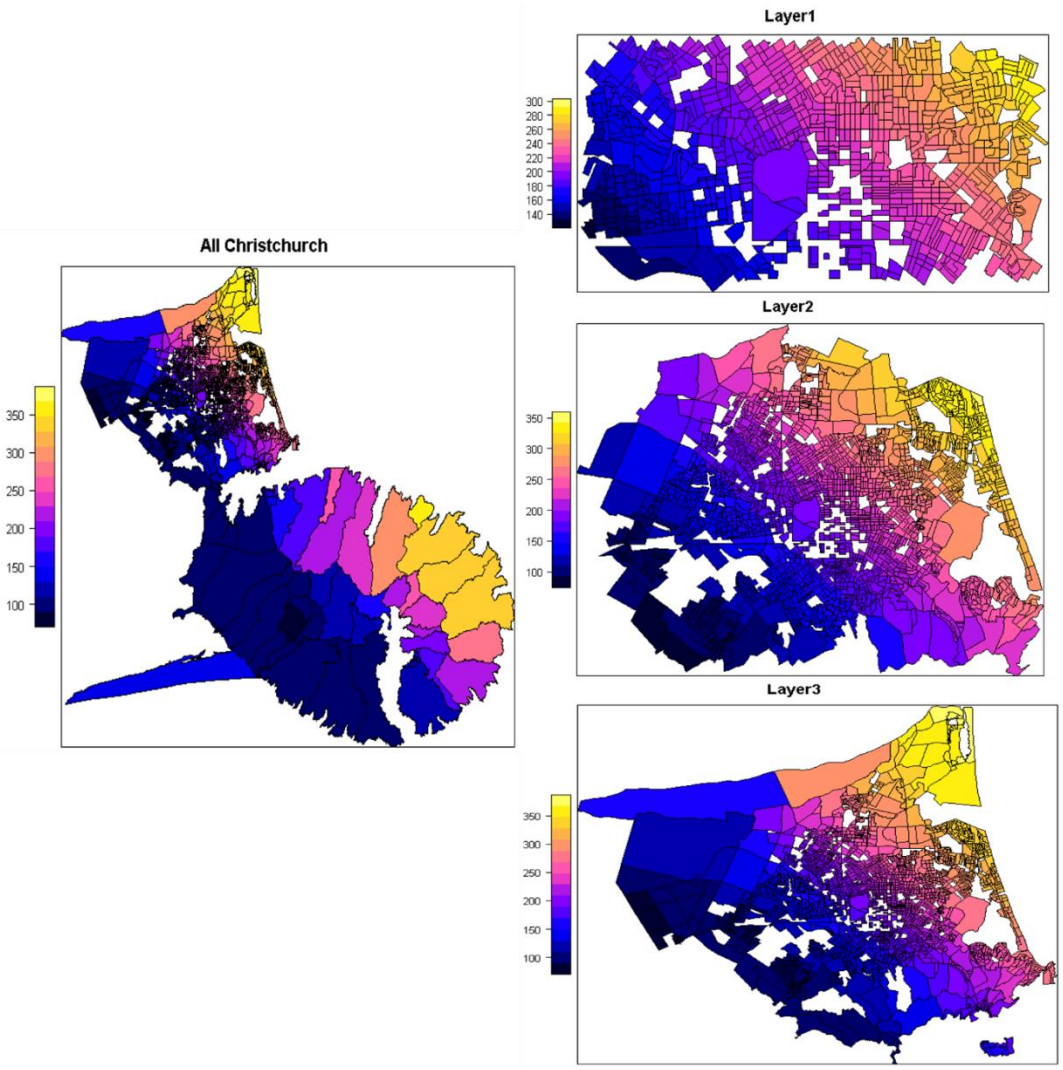


Figure 5-16 Spatial trends of the response variable y for all Christchurch meshblocks and three different layers of the Christchurch meshblocks.

After selecting 1000 samples from each sampling scheme, the variance of the HT estimator was simulated:

$$\hat{V}(\hat{T}_{HT}) = \frac{1}{1000}\sum_{i=1}^{1000}(\hat{T}_{HT_i} - T)^2 \tag{5.3}$$

where $\hat{T}_{HT\_i}$ is the total value of the response variable, which is estimated from the i^{th} iteration and T is the true total value of the response variable.

In this study, the total value of the response variable ($T = \sum_i y_i$) is known for each layer.

Regarding the simulated $\hat{V}(\hat{T}_{HT})$, $Deff$ was estimated for each sampling design using the following equation:

$$Deff = \frac{\hat{\mathrm{V}}\left(\hat{\mathrm{T}}_{\mathrm{HT}-Complex\ Survey}\right)}{\hat{\mathrm{V}}\left(\hat{\mathrm{T}}_{\mathrm{SRS}}\right)}. \tag{5.4}$$

The results of the simulation study for estimating the response variable y are shown in Table 5-4. The average of ζ (according to Equation (2.22)) among all 1000 replicates ($\hat{\mu}(\zeta)$) is also reported in Table 5-4 as an index of spatial balance created by each sampling design.

Table 5-4 shows that for all sampling fractions and in all considered layers, the $Deff$ related to the spatially balanced sampling methods are smaller than 1. This shows that the spatially balanced sampling methods estimated the total value of the response variable ($\hat{T}_{HT}$) with a variance that was smaller than the variance of the SRS method. Therefore, the spatially balanced sampling methods are more precise than SRS in terms of estimating the population parameters. Table 5-4 shows that there is no remarkable difference between results in the different layers considered. It confirms that the spatially balanced sampling methods can select precise samples from populations with different levels of density. Findings from Table 5-4 indicate that the BAS-Frame method can work as precisely as other spatially balanced sampling methods. Comparing two scenarios (adding or removing random points) for implementing the BAS-Frame method, it was found that the removal of random points results in samples which are more spatially balanced. In this example, the population units had the same inclusion probability; therefore removing points randomly did not change the probability of selection of the population units. The values of $\hat{\mu}(\zeta)$ indicate that LPM1 provides more spatially balanced samples when compared to other methods.

Table 5-4 Achieved values of $\hat{\mu}(\zeta)$ and $Deff$ for estimating the response variable among all Christchurch and three different layers using 1000 simulated samples with different sampling fractions using four different sampling schemes.

	Design	Sampling fraction					
		1%		2%		3%	
		$\hat{\mu}(\zeta)$	Deff	$\hat{\mu}(\zeta)$	Deff	$\hat{\mu}(\zeta)$	Deff
Layer1	SRS	0.35	/	0.34	/	0.34	/
	LPM	0.10	0.19	0.09	0.12	0.08	0.08
	GRTS	0.15	0.31	0.13	0.15	0.13	0.17
	BAS(removing random points)	0.11	0.14	0.10	0.09	0.10	0.06
	BAS(adding random points)	0.13	0.24	0.12	0.13	0.12	0.14
Layer2	SRS	0.34	/	0.36	/	0.37	/
	LPM	0.09	0.09	0.10	0.06	0.10	0.04
	GRTS	0.15	0.16	0.13	0.07	0.14	0.06
	BAS(removing random points)	0.09	0.05	0.11	0.06	0.12	0.04
	BAS(adding random points)	0.14	0.19	0.13	0.09	0.15	0.06
Layer3	SRS	0.36	/	0.38	/	0.39	/
	LPM	0.11	0.10	0.10	0.06	0.10	0.07
	GRTS	0.16	0.20	0.14	0.14	0.15	0.10
	BAS(removing random points)	0.09	0.08	0.12	0.09	0.14	0.10
	BAS(adding random points)	0.16	0.16	0.17	0.10	0.17	0.12
All Christchurch	SRS	0.38	/	0.36	/	0.36	/
	LPM	0.11	0.14	0.09	0.07	0.10	0.06
	GRTS	0.16	0.19	0.14	0.14	0.15	0.13
	BAS(removing random points)	0.13	0.10	0.13	0.07	0.12	0.07
	BAS(adding random points)	0.16	0.20	0.17	0.17	0.17	0.16

Table 5-4 Continued

	Design	Sampling fraction			
		4%		5%	
		$\hat{\mu}(\zeta)$	Deff	$\hat{\mu}(\zeta)$	Deff
Layer1	SRS	0.34	/	0.33	/
	LPM	0.08	0.09	0.09	0.08
	GRTS	0.13	0.18	0.13	0.18
	BAS(removing random points)	0.11	0.06	0.12	0.07
	BAS(adding random points)	0.13	0.15	0.13	0.15
Layer2	SRS	0.35	/	0.37	/
	LPM	0.11	0.03	0.11	0.02
	GRTS	0.14	0.04	0.14	0.04
	BAS(removing random points)	0.14	0.03	0.12	0.03
	BAS(adding random points)	0.18	0.07	0.13	0.05
Layer3	SRS	0.37	/	0.38	/
	LPM	0.11	0.04	0.11	0.04
	GRTS	0.15	0.08	0.15	0.05
	BAS(removing random points)	0.15	0.08	0.15	0.08
	BAS(adding random points)	0.16	0.08	0.17	0.08
All Christchurch	SRS	0.37	/	0.38	/
	LPM	0.11	0.05	0.11	0.07
	GRTS	0.15	0.09	0.15	0.07
	BAS(removing random points)	0.14	0.07	0.14	0.06
	BAS(adding random points)	0.17	0.11	0.18	0.11

5.3.3 Application of Spatially Balanced Sampling Methods on Real Data

Previous sections have explained how spatially balanced sampling methods can be used to spread the sampling units over a finite population in a household survey. Then, the spatial and statistical characteristics of the introduced methods were evaluated using either artificial datasets or artificial response variables. In this section, the efficiency of spatially balanced sampling methods will be investigated through conducting a simulation study on a real dataset

of meshblocks of the Canterbury region in New Zealand. The dataset is available on the Stats NZ website (Stats NZ, 2013a). This simulation study again supposes that the meshblocks are the ultimate population units and will select them using equal probability sampling schemes.

In addition to the longitude and latitude of the centre of each meshblock, the dataset contains nine attributes relevant to each meshblock. The variables are the following:

1. male: number of males
2. female: number of females
3. Māori: number of Māori people
4. child: number of people who are 0 to 14 years old
5. young: number of people who are 15 to 64 years old
6. adult: number of people who are more than 65 years old
7. unemployed: number of unemployed people
8. employed: number of employed people
9. income: average income of households.

Moran's I for the above variables based on the rook's definition of a neighbourhood are shown in Table 5-5.

Table 5-5 Moran's I for the response variables among meshblocks in Canterbury region.

Variable Name	Moran's I
Male	0.26
Female	0.28
Māori	0.29
Child	0.26
Young	0.28
Adult	0.21
Unemployed	0.24
Employed	0.27
Income	0.34

As Table 5-5 shows, for all the considered response variables, there is a moderate positive spatial autocorrelation.

In this simulation study, six spatially balanced sampling designs were applied to select the samples: LPM, GRTS, SCPS, CUBE, doubly balanced sampling (here called LCUBE)

and BAS-Frame. Two scenarios were considered for the BAS-Frame method: adding and removing random points. SRS was also used as a benchmark for comparing the spatially balanced sampling methods. For selecting spatially balanced samples using the CUBE and LCUBE methods, the coordinates of the centres of the meshblocks were considered as balanced variables. As in other simulation studies that have been conducted in this thesis, a range of five different sampling fractions was considered: 1, 2, 3, 4 and 5% of the meshblocks.

After selecting 1000 samples for each sampling method and calculating the variance of the HT estimator for estimating the mean of each response variable, the $Deff$ relevant to each sampling design was calculated using Equation (2.12). Results of the simulation study for estimating the mean of the response variables for different sampling fractions are reported in Table 5-6.

Table 5-7 also reports the average of $Deff$ (over all variables) for each sampling design.

Table 5-6 Estimated Deff relevant to each sampling design for estimating the mean of the considered response variables.

Sampling Fraction	Response variables	Sampling Design					
		BAS-Frame	LPM	GRTS	SCPS	CUBE	LCUBE
1%	Male	0.89	0.67	0.64	074	0.72	0.89
	Female	0.76	0.67	0.82	0.85	0.76	0.93
	Māori	0.81	0.87	0.92	0.75	0.93	0.88
	Child	0.89	0.88	0.77	0.78	0.77	0.64
	Young	0.89	0.73	0.67	0.95	0.91	0.99
	Adult	0.88	0.68	0.75	0.69	0.74	0.89
	Unemployed	0.85	0.76	0.83	0.46	0.77	0.53
	Employed	0.65	0.57	0.68	0.98	0.96	0.86
	Income	0.96	0.65	0.97	0.89	0.87	0.95
2%	Male	0.96	0.94	0.84	0.92	0.99	0.82
	Female	0.92	0.89	0.89	0.88	0.87	0.95
	Māori	0.79	0.46	0.71	0.78	0.86	0.93
	Child	0.98	0.95	0.98	0.96	0.96	0.97
	Young	0.83	0.75	0.76	0.78	0.78	0.80
	Adult	0.55	0.53	0.72	0.62	0.62	0.92
	Unemployed	0.37	0.86	0.32	0.52	0.80	0.72
	Employed	0.93	0.86	0.88	0.98	0.96	0.81
	Income	0.75	0.65	0.86	0.85	0.76	0.87
3%	Male	0.93	0.86	0.71	0.87	0.92	0.68
	Female	0.92	0.91	0.76	0.73	0.92	0.75
	Māori	0.78	0.86	0.62	0.87	0.77	0.84
	Child	0.76	0.75	0.73	0.75	0.89	0.62
	Young	0.82	0.75	0.83	0.96	0.95	0.71
	Adult	0.63	0.58	0.75	0.52	0.95	0.75
	Unemployed	0.86	0.82	0.38	0.94	0.92	0.85
	Employed	0.89	0.82	0.73	0.96	0.93	0.76
	Income	0.76	0.64	0.76	0.84	0.78	0.68

Table 5-6 Continued

Sampling Fraction	Response variables	Sampling Design					
		BAS-Frame	LPM	GRTS	SCPS	CUBE	LCUBE
4%	Male	0.87	0.77	0.52	0.84	0.79	0.59
	Female	0.89	0.76	0.56	0.88	0.89	0.79
	Māori	0.96	0.90	0.94	0.84	0.68	0.78
	Child	0.77	0.71	0.78	0.89	0.87	0.54
	Young	0.95	0.58	0.50	0.73	0.76	0.46
	Adult	0.76	0.78	0.96	0.89	0.91	0.92
	Unemployed	0.75	0.83	0.71	0.52	0.49	0.72
	Employed	0.98	0.55	0.52	0.85	0.96	0.69
	Income	0.67	0.65	0.69	0.75	0.92	0.96
5%	Male	0.44	0.52	0.59	0.89	0.53	0.84
	Female	0.65	0.51	0.72	0.85	0.52	0.67
	Māori	0.83	0.73	0.95	0.98	0.78	0.93
	Child	0.74	0.49	0.85	0.86	0.73	0.90
	Young	0.67	0.53	0.56	0.66	0.54	0.79
	Adult	0.77	0.76	0.92	0.92	0.95	0.73
	Unemployed	0.45	0.54	0.50	0.46	0.56	0.90
	Employed	0.65	0.62	0.65	0.82	0.63	0.76
	Income	0.69	0.68	0.68	0.75	0.93	0.95

Table 5-7 Average of Deff on all response variables relevant to each sampling design for estimating the average value of the considered response variables.

Sampling Fraction	BAS-Frame (adding random points)	BAS-Frame (removing random points)	LPM	GRTS	SCPS	CUBE	LCUBE
1%	0.84	0.78	0.72	0.78	0.79	0.83	0.84
2%	0.79	0.78	0.77	0.77	0.81	0.84	0.87
3%	0.72	0.75	0.78	0.70	0.83	0.89	0.74
4%	0.75	0.71	0.66	0.70	0.80	0.75	0.77
5%	0.65	0.65	0.60	0.71	0.80	0.69	0.83

Findings from Table 5-6 and Table 5-7 show that, for each evaluated sampling design, and at each level of the sampling fraction, the $Deff$ is less than 1. This means that, as expected, in household sampling surveys, spatially balanced sampling designs can provide more precise estimates than SRS. The CUBE and LCUBE methods provide slightly higher

$Deff$ compared with other spatially balanced sampling designs. Therefore, these methods are not recommended for selecting a spatially balanced sample in a household survey. Of all the evaluated methods, again, LPM provides smallest $Deff$.

In order to compare the evaluated methods in terms of spreading the sample meshblocks over the Canterbury region, the average of ζ (Equation (2.22)) among all 1000 replicates ($\hat{\mu}(\zeta)$) was calculated for each sampling design and for all sampling fractions. The results are shown in Table 5-8.

Table 5-8 Average of ζ among all 1000 replicates for the evaluated sampling design.

Sampling Fraction	Sampling Design							
	BAS-Frame (adding random points)	BAS-Frame (removing random points)	LPM	GRTS	SCPS	CUBE	LCUBE	SRS
1%	0.19	0.19	0.13	0.18	0.14	0.38	0.15	0.53
2%	0.20	0.18	0.13	0.18	0.14	0.37	0.15	0.48
3%	0.19	0.19	0.14	0.19	0.14	0.39	0.16	0.44
4%	0.19	0.20	0.14	0.18	0.14	0.39	0.16	0.44
5%	0.21	0.20	0.14	0.18	0.15	0.38	0.16	0.43

Results of $\hat{\mu}(\zeta)$ shown in Table 5-8 indicate, as expected, that the spatially balanced sampling methods are more powerful than SRS in spreading the sampling meshblocks over the region of Canterbury. The CUBE method did not provide samples as spatially balanced as other spatially balanced sampling methods. The most spatially balanced samples were selected by LPM. As shown in Table 5-8, the $\hat{\mu}(\zeta)$ associated with BAS-Frame and with removing random points is either equal or less than the $\hat{\mu}(\zeta)$ associated with BAS-Frame with adding random points. Therefore, in this study the use of the BAS-Frame method with removing random points is suggested for selecting spatially balanced samples when the population units have equal probabilities of being selected.

5.4 Implementation of Spatially Balanced Sampling Methods on Stratified Populations in Household Surveys

Stratification of the target population prior to the sample selection is a common technique used in designing a sampling household survey. Providing unbiased estimates of parameters

of interest for important groups in the population, making sure that important groups in the population have proper representation in the sample, and increasing the precision of the estimates at the national level are three main reasons for stratifying the population in household surveys. To achieve these goals, the target population in household surveys might be stratified by either the geographic or demographic characteristics of the units.

In Chapter 4, the application of the BAS method for selecting samples from a continuous stratified population was discussed. The application of the spatially balanced sampling methods, specifically the BAS-Frame technique, on discrete stratified populations will be studied in this section. The study will use two different kinds of stratified populations:

a) a population that is stratified according to the geographical characteristics (e.g., rural or urban), and
b) a population that is stratified using demographic characteristics (e.g., sex, age).

Also, two different sample allocation methods (proportionate and disproportionate allocation) will be considered.

5.4.1 BAS-Frame Technique for a Stratified Population

To select a spatially balanced sample from a discrete stratified population using the BAS-Frame technique, two options can be suggested.

In the first option, a primary frame (and consequently a regular frame) is created on the entire region of the population of interest in the first step. In the second step, the strata boundaries are defined on the created primary frame, and then sampling units are selected by applying the BAS method in each created stratum independently. Alternatively, in the second option, the population is stratified first, and then the BAS-Frame technique is applied in each stratum, independently. The steps of the sample selection process using the two options are illustrated in Figure 5-18 and Figure 5-19, respectively. These figures aim to select a spatially balanced sample from a population consisting of 64 units that are stratified into two strata ("x" and "o"). The geographical locations of the units in the population are illustrated in Figure 5-17.

x	o	o	o	x	o	x	o
o	x	o	o	o	x	o	o
x	x	o	o	x	o	x	x
o	o	x	x	x	x	x	o
o	x	x	o	o	x	o	x
x	o	x	x	x	o	o	o
o	x	x	o	o	o	x	x
x	x	o	x	x	o	o	x

Figure 5-17 A population consisting of 64 units which are stratified into two strata ("x" and "o").

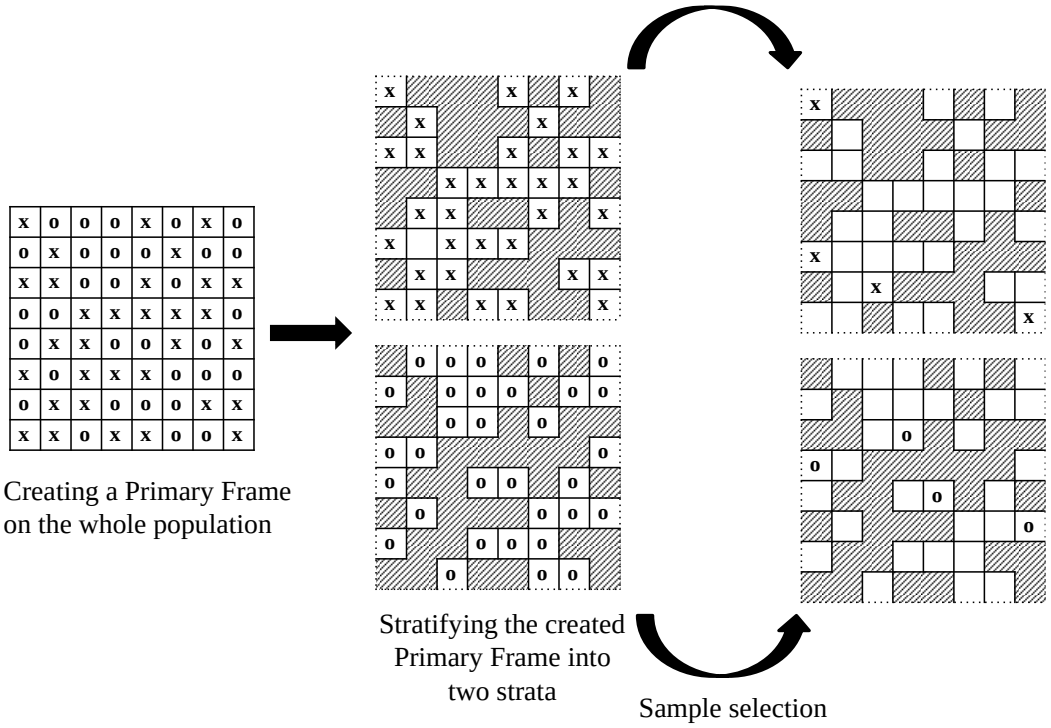


Figure 5-18 The first option for sample selection from a stratified population using the BAS-Frame technique.

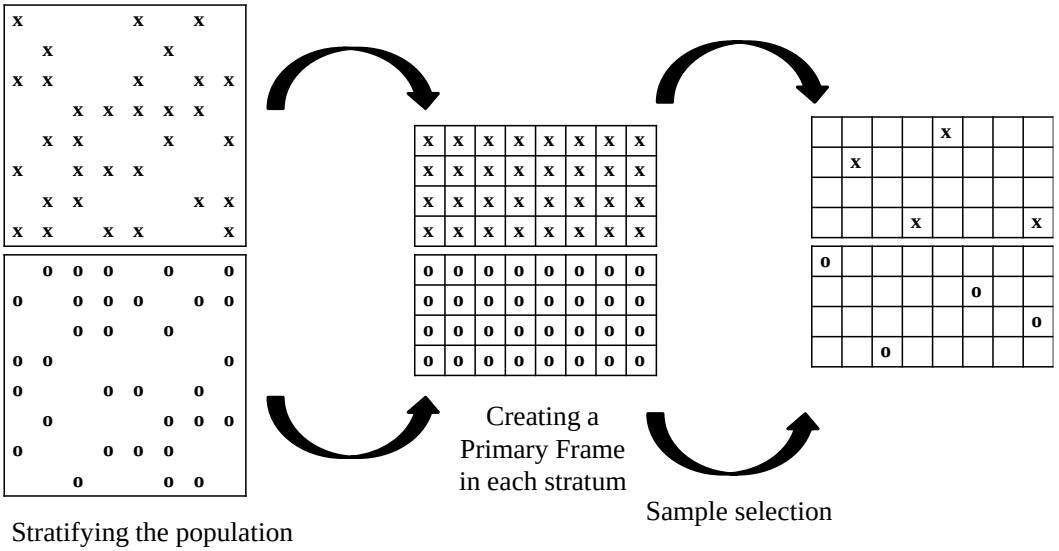


Figure 5-19 The second option for sample selection from a stratified population using the BAS-Frame technique.

Although both options select spatially balanced samples, the efficiency of the second option is higher than the first option. In fact, in the first option, the Primary Frame corresponding to each stratum contains a number of empty boxes (hatched boxes in Figure 5-18), and therefore this may lengthen the sample selection process.

In this study, the second option is used for selecting spatially balanced samples from the stratified population using the BAS-Frame technique.

5.4.2 Spatially Balanced Sampling Methods When the Population is Stratified Geographically

Stratifying the population of interest into urban and rural areas is a common task in almost all household sampling surveys. With this stratification, the survey costs can be controlled by implementing different sampling schemes in each stratum. In general, as travelling costs in urban areas are lower, selecting well-spread samples may be desirable. In contrast, as travelling costs in rural areas are usually high, it might be preferable to select sampling units near to each other rather than spreading them over the population. Hence, spatially balanced sampling methods might not be a practical option for selecting samples from rural areas.

This subsection aims to investigate the travelling costs in rural and urban areas when sampling units are selected using spatially balanced sampling methods. For this, a simulation study was conducted on meshblocks of Ashburton town in New Zealand. Meshblocks in this town are categorized into five geographical levels: main urban, secondary urban, minor urban, rural centre, and rural areas. Using this information, the "urban" stratum was generated by combining main urban, secondary urban and minor urban meshblocks. The combination of rural centre and rural area meshblocks, was considered as the "rural" stratum.

After defining rural and urban strata, two different spatially balanced sampling designs (LPM and BAS-Frame technique) and a non-spatially balanced sampling method (SRS) were employed to select sample meshblocks from each stratum by considering equal sampling fractions. The sampling process was repeated 1000 times for three different sampling fractions (10, 15 and 20%).

In the dataset, the only available information for modelling travel cost is the geographical coordinates of the centre of meshblocks in the population frame. Therefore, by assuming that travelling cost is affected only by travelling distance, the length of the path among selected meshblocks is considered as an index to define the travelling costs for that sample. The distance in this study was computed using the travelling salesperson problem (TSP) (Hahsler & Hornik, 2007). TSP's goal is to find the shortest path that visits each sampled meshblock once and then returns to the starting meshblock.

After selecting 1000 samples, the average of the shortest distance for each sampling method ($\bar{d}$) was calculated using

$$\bar{d} = \frac{1}{1000} \sum_{i=1}^{1000} d_i \tag{5.5}$$

where d_i is the shortest distance, which is determined with the help of TSP, for the i^{th} iteration. The values d_i and $\bar{d}$ are expressed in kilometres (km).

The calculated values of $\bar{d}$ for the evaluated sampling methods in the urban and rural strata and for three different sampling fractions are shown in Table 5-9.

*Table 5-9 Calculated $\bar{d}$(*km*) among 1000 repetitions for the evaluated sampling methods in urban and rural strata.*

Sampling Fraction	Urban Stratum			Rural Stratum		
	LPM	BAS-Frame	SRS	LPM	BAS-Frame	SRS
10%	79,960	79,753	77,962	296,534	295,660	288,152
15%	90,363	89,410	87,590	359,415	358,411	344,976
20%	97,125	97,927	95,387	406,095	401,898	389,749

As Table 5-9 shows, for all sampling fractions, information from meshblocks in rural areas need to be collected by travelling longer distances than in urban areas. For both urban and rural areas, the largest distances are provided when sample meshblocks were selected by the LPM method. This suggests that LPM spreads the sample meshblocks over the Ashburton township better than the two other considered methods.

As for other indexes, in this simulation study the relative distance related to each spatially balanced sampling design (r_{SBS}) was calculated using

$$r_{SBS} = \frac{\bar{d}_{SBS} - \bar{d}_{SRS}}{\bar{d}_{SRS}} \times 100\% \tag{5.6}$$

where $\bar{d}_{SBS}$ and $\bar{d}_{SRS}$ are calculated based on Equation (5.5) for the spatially balanced sampling methods and SRS, respectively.

Values of r_{SBS} greater than 0 show that the travelling distance for visiting sampling units selected by the spatially balanced sampling methods is longer than the required travelling distance to visit sampling units selected by SRS. The calculated relative distances corresponding to the LPM and BAS-Frame techniques, for urban and rural strata, and for three different sampling fractions, are summarized in Table 5-10.

Table 5-10 Calculated relative distance corresponding to LPM and BAS-Frame technique, for urban and rural strata, and for three different sampling fractions.

Sampling Fraction	Urban Stratum		Rural Stratum	
	LPM	BAS-Frame	LPM	BAS-Frame
10%	2.56%	2.30%	2.91%	2.61%
15%	3.17%	2.08%	4.19%	3.89%
20%	1.82%	2.66%	4.19%	3.12%

Findings from Table 5-10 confirm that using spatially balanced sampling methods in selecting samples from both urban and rural areas can lead to the longest travelling distance to visit all the selected units compared to the situation where samples are selected by SRS. However, the values of r_{SBS} related to the rural stratum are higher than the corresponding values for the urban stratum. It shows that using spatially balanced sampling methods in rural areas is more affected by increased travelling costs than when it is used in urban areas.

To decrease the travelling path and consequently reduce the travelling costs, one may suggest selecting a less spatially balanced sample from the rural areas. This can be met by modifying the BAS-Frame method in such a way that the created boxes include more than one meshblock. This modification of the BAS-Frame will be studied in more detail in Chapter 6.

5.4.3 Spatially Balanced Sampling When the Population is Stratified Demographically

In addition to geographical stratifications, populations in household sampling surveys might be stratified using socio-economic and/or demographic auxiliary variables. Although the advantages of the stratified sampling method have been reported widely in a number of studies, there have been some limitations in applying this method in household surveys (Turner, 2003; Lynn, 2019). Besides selecting the relevant demographic stratification variables, finding an efficient way to stratify the population is one of the biggest challenges in using the stratified sampling method in household sampling surveys.

This subsection aims to investigate whether the stratified sampling method can be substituted by a spatially balanced sampling method when the population in a household survey is stratified by socio-economic auxiliary variables. This question will be studied in this subsection by focusing on two different situations:

a) when the stratification is used only to guarantee that a certain group in the population has proper representation in the sample, and
b) when the survey is a multi-objective survey where finding the relevant stratification variables may not be possible.

These two situations were studied through simulation on meshblocks of Christchurch city. For simplicity, the population of meshblocks was stratified using only one stratification variable.

Māori are deemed to be a group of high importance for most household sampling surveys conducted by Stats NZ. Hence, Stats NZ tries to select an adequate representation of this group in its household samples by targeting areas of high Māori population density. Usually this is done by considering Māori population density as one of the stratification variables. Here, to make the simulation studies more similar to the household sampling surveys of Stats NZ, the density of the Māori population is considered as the stratification variable.

In this study, according to the proportion of Māori in meshblocks, the meshblocks in Christchurch city are stratified into two strata:

1) stratum with high Māori population density ("high Māori"), and
2) stratum with low Māori population density ("low Māori").

In this study, the high Māori stratum includes meshblocks that have more than 12 percent Māori. In contrast, the proportion of Māori in meshblocks located in the low Māori stratum is less than 12 percent. The total number of meshblocks in Christchurch city that are located in the high Māori stratum is almost 4 times greater than the number of meshblocks located in the low Māori stratum.

The details of the simulation studies related to each of the two situations mentioned above are explained next.

Situation (a): when the stratification is used only to guarantee that a certain group in the population has proper representation in the sample

In some practical cases, the stratification process is done only to ensure that all target groups – especially those that represent a small proportion of the population – are represented in the sample appropriately. In these situations, the stratification is not aimed at providing separate estimates for these groups. Chapter 4 of this thesis showed that in cases of continuous populations, the BAS method can perform as well as the stratified sampling method with proportional allocation; in other words, the population does not need to be stratified explicitly. As explained before, this is because the number of sampling units selected by BAS from a specific part of a continuous population is proportional to the area of that part.

The simulation study in this subsection aims to understand whether the BAS-Frame method can be used as an alternative method to the stratified sampling method with proportional allocation for selecting samples from a stratified discrete population.

In the simulation study, 1000 samples of different sizes (1, 2, 3, 4, and 5% of the total number of meshblocks) were selected from the meshblocks of Christchurch city without attention to the boundaries of the defined strata (high Māori and low Māori). Samples were selected using two spatially balanced sampling methods (LPM and BAS-Frame) and a non-spatially balanced sampling method (SRS). For each selected sample the numbers of meshblocks located in the low Māori stratum and high Māori stratum were counted. Let m_{hi} $(h \in \{low\ M\bar{a}ori, high\ M\bar{a}ori\},\ i = 1, \dots, 1000)$ be the number of selected meshblocks within stratum h at the i^{th} iteration.

Using the stratified sampling method with proportional allocation ensures that the number of selected meshblocks in the high Māori stratum was almost 4 times greater than the number of selected meshblocks in the low Māori stratum. Therefore, comparing the values of m_{hi} with the expected value according to the proportional allocation will reveal to what extent the performance of the evaluated designs is close to the performance of the proportional stratified sampling method. The average values and variance of m_{hi} among all iterations for all evaluated designs and all sample sizes are shown in Table 5-11. The numbers of allocated meshblocks to strata according to the stratified sampling method with proportional allocation are in bold in Table 5-11.

As seen in Table 5-11, for all the sample sizes, the average number of selected meshblocks (m_{hi}) in each stratum obtained by spatially balanced sampling methods satisfactorily matches the expected values associated with a proportional stratified sampling method. This means that the samples selected by the spatially balanced sampling methods consist of meshblocks with different Māori population density. The results confirm that implementing a spatially balanced sampling method in irregular discrete populations can perform as well as proportional stratified sampling.

Table 5-11 The average and variance of m_{hi} among all iterations for all evaluated designs and all sample sizes.

Sampling Design	Sample size	Average of m_{hi}		Variance of m_{hi}
		low Māori stratum	high Māori stratum	
stratified	35	**10**	**25**	Not Applicable
LPM		10.3	24.97	6.03
BAS-Frame		10.5	24.95	5.91
SRS		9.01	26.19	6.94
stratified	69	**20**	**49**	Not Applicable
LPM		19.91	49.09	13.53
BAS-Frame		19.85	49.05	10.01
SRS		19.01	48.39	13.5
stratified	104	**30**	**74**	Not Applicable
LPM		29.78	74.22	18.12
BAS-Frame		29.57	74.43	15.98
SRS		29.53	74.47	19.54
stratified	138	**40**	**98**	Not Applicable
LPM		39.92	98.08	25.02
BAS-Frame		39.85	98.15	20.68
SRS		39.34	98.66	25.29
stratified	173	**50**	**123**	Not Applicable
LPM		49.67	123.63	31.52
BAS-Frame		49.87	123.13	25.64
SRS		49.24	123.76	34.49

Furthermore, it was found that of the three evaluated methods in the current study (i.e., LPM, BAS-Frame and SRS), the variance of m_{hi} related to the BAS-Frame method is the smallest (Figure 5-20). This implies that BAS-Frame was more stable than other methods in different repetitions. This finding makes the BAS-Frame method a desirable technique for selecting samples from a stratified population, from a practical point of view.

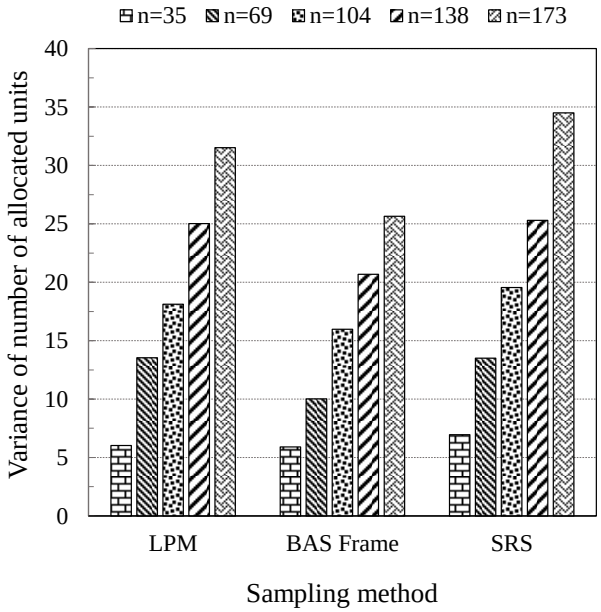


Figure 5-20 Variance of m_{hi} among all iterations for all evaluated designs and all sample sizes.

Situation (b): when the survey is a multi-objective survey where finding the relevant stratification variables may not be possible

It is well understood, from the sampling literature, that the stratified sampling method increases the precision of estimates when the population is stratified using a variable that is correlated with the characteristic under study (Cochran, 1977). However, the problem of stratifying the population becomes complicated when there are many target variables which are not necessarily related to each other (D'Orazio & Catanese, 2016). In this situation, stratification on a specific variable may not be efficient for all the target variables. This is a general phenomenon in household surveys that aim to estimate several population characteristics by running a single survey. In these situations, one may suggest selecting samples by implementing spatially balanced sampling methods instead of the stratified sampling method. This idea is investigated in this subsection through a simulation study. The simulation study aims to investigate whether using the spatially balanced sampling methods in a multi-purpose survey can provide more precise estimates than the stratified sampling method.

As in the previous simulation study, the meshblocks in Christchurch city are stratified into high Māori and low Māori strata. Let a survey be planned for purposes of estimating the following characteristics:

- total number of people who bike to work,
- total number of people who have a partner,
- total number of females with no alive born children,
- total number of people who can speak te reo Māori (the Māori language) , and
- total number of people who are involved in unpaid household work.

Of the five considered target variables, it could be assumed that "people who can speak te reo Māori" is the only variable that might have a correlation with the stratification variable (high/low Māori). On the basis of the available dataset, there is a positive correlation between the "people who can speak te reo Māori" and the stratification variable (i.e., approximately 11%). The other variables do not have any correlation with the stratification variable.

Sample meshblocks in this study were selected using SRS, LPM and BAS-Frame methods separately in each created stratum. Also, LPM and BAS-Frame were used to select spatial samples without attention to the boundaries of the strata. The sample selection was repeated 1000 times and for five different sample sizes (1, 2, 3, 4, and 5% of the total number of meshblocks). For simplicity, the same sampling fraction was used in each stratum.

After selecting samples, the variance of the HT estimator for estimating the characteristics of interest was estimated for each evaluated sampling method. In order to compare the evaluated designs, the ratio of variance related to each spatially balanced sampling design was calculated by dividing the simulated variance of that spatial design by the simulated variance of the stratified SRS.

$$\text{Ratio of Variance} = \frac{\text{Variance of spatially balanced sampling design}}{\text{variance of stratified SRS}}. \qquad (5.7)$$

The ratio of variances related to each characteristic and for all sample sizes are reported in Table 5-12. Trends of ratio of variances related to each target variable among the different sample sizes are shown in Figure 5-21.

Table 5-12 The ratio of variance related to each target variables and for all considered sample sizes.

Target variable	Sample design	Sampling fraction				
		1%	2%	3%	4%	5%
Bike user	Stratified LPM	0.81	1.02	0.97	0.89	0.97
	Stratified BAS	0.99	1.17	1.07	0.92	1.03
	LPM	0.84	0.89	0.80	0.99	1.03
	BAS	0.87	1.00	1.09	1.10	1.17
Māori speaker	Stratified LPM	0.99	1.04	0.93	1.02	0.90
	Stratified BAS	0.96	0.97	0.83	0.85	0.96
	LPM	1.10	1.08	1.09	1.09	1.10
	BAS	0.96	0.99	1.02	1.00	1.14
Females with no alive born children	Stratified LPM	0.71	0.30	0.67	0.27	0.88
	Stratified BAS	0.42	0.31	0.63	0.60	0.71
	LPM	0.62	0.36	0.66	0.60	0.70
	BAS	0.66	0.35	0.67	0.43	0.75
People involved in unpaid household work	Stratified LPM	1.01	1.03	0.70	0.65	0.76
	Stratified BAS	0.74	1.18	0.76	0.80	0.95
	LPM	0.32	0.90	0.56	0.93	0.67
	BAS	0.54	0.90	0.49	0.90	0.25
People with partner	Stratified LPM	0.51	1.00	0.69	1.10	0.59
	Stratified BAS	0.90	0.93	0.70	1.10	0.69
	LPM	0.35	0.57	0.39	0.40	0.51
	BAS	0.40	0.31	0.32	0.71	0.45

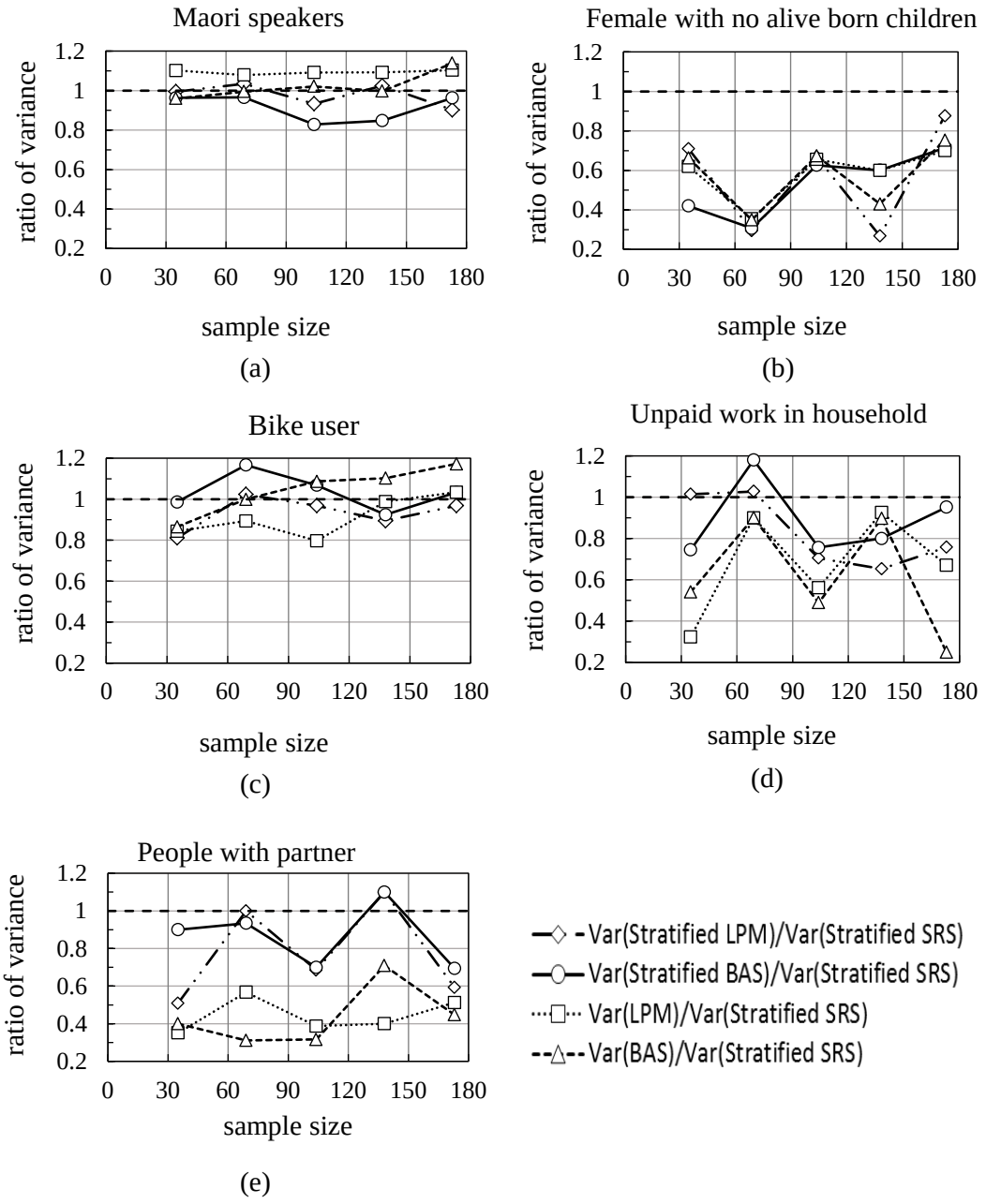


Figure 5-21 Trends of ratio of variances related to each target variable among the different sample sizes.

The gains from using the stratification technique can be seen clearly in estimating the total number of people who can speak te reo Māori (Figure 5-21a). In this case, the ratio of the variance related to the stratified LPM and the stratified BAS-Frame are consistently smaller than the ratio of the variance related to LPM and BAS-Frame for all considered

sample sizes, respectively. This emphasises that stratifying the population with an auxiliary variable that is correlated with the target variable can increase the precision of the estimates.

Figure 5-21a also shows that the ratio of variances related to both the stratified LPM and the stratified BAS-Frame for almost all considered sample sizes are less than 1. This indicates that spatially stratified sampling methods have smaller variance than the non-spatially balanced sampling method.

However, as Figure 5-21b–e show, the stratified techniques did not make a contribution in decreasing the variances of the HT estimator in estimating the other characteristics of interest. On taking a closer look at the results corresponding to these variables, it is clear that the estimated variances related to the LPM and BAS-Frame are smaller than the relevant values related to the stratified LPM and the stratified BAS-Frame, respectively.

Results achieved from the simulation study show that, in cases of multipurpose surveys where there is an interest in many variables, stratification based on one socio-economic variable that is not related to all target variables may not provide the best stratification for the others. In these cases, selecting samples based on their geographical coordinates provided more precise estimates than the stratified sampling method. Therefore, this thesis suggests using the spatially balanced sampling methods in multipurpose surveys where it is not possible to optimise the design based on all target variables.

5.5 Conclusions

Sampling units in household surveys are usually selected by employing multistage sampling designs in such a way that housing units (or households) are sampled within the selected area units through several stages. These conventional sampling methods generally do not take into account the spatial dependency that can exist among population units. Recent advances in geographical technologies (i.e., GIS and GPS) have provided opportunities to apply spatial sampling methods in household sampling surveys.

Although spatially balanced sampling methods have been initially designed for environmental studies, the study showed that they have potential to be used in household surveys. While spatially balanced sampling methods in environmental studies deal with continuous populations, spatially balanced samples in household surveys need to be selected

from target populations that typically consist of a finite number of discrete units. Thus, in this chapter, a new modification of the BAS method, the BAS-Frame method, was developed.

Results from simulations showed that the BAS-Frame method is able to select spatially balanced samples as well as other spatially balanced sampling methods.

In the second part of this chapter, the BAS-Frame method along with spatially balanced sampling methods were examined in the process of selecting samples from a list of meshblocks in the region of Canterbury, New Zealand. The outcomes indicated that spatially balanced sampling methods provided more precise estimates of the population characteristics when compared to the SRS method in spite of relatively poor spatial autocorrelation for the considered response variables. It was found that LPM has the best performance among all the methods evaluated in this study.

The third part of the chapter investigated the application of the BAS-Frame method in populations that are stratified either geographically or demographically. In household surveys, a common geographical stratification variable is urban/rural area. In order to investigate the undesirable effect of spatially balanced sampling on the survey cost in each urban and rural area, a simulation was performed on a population that was stratified into rural and urban strata. Results of the simulation study confirmed that in comparison to SRS, using the spatially balanced sampling in a rural stratum may increase the travel costs, while travel cost increase is not marked in the urban stratum.

Regarding the use of geographic variables in the stratification process, this chapter focused on two different situations: when the stratification is used only to guarantee that sampling units are spread evenly over the population and when a population needs to be stratified for running a multi-objective survey. Findings derived from the simulation studies showed that, for the first situation, and in cases where the same sampling fraction is used in all strata, it is not necessary to stratify the population explicitly. In these cases, a BAS-Frame technique can be used as an alternative method to the stratified sampling method with proportional allocation. Finding a suitable stratification variable that is related to all target variables is one difficulty in using the stratified sampling in multi-objective surveys. One suggestion to address this difficulty is using spatially balanced sampling methods. This suggestion was investigated through a simulation study. Results of the simulation study showed that the spatially balanced sampling methods provided more precise estimates than

the stratified sampling method when the survey is a multi-objectives survey and finding a relevant stratification variable is not possible.

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Chapter 6 *Properties of Sampling Frames for Spatial Sampling in Household Surveys*

6.1 Introduction

To conduct a probability sample a list of target units should be available from which the samples are drawn. Such a list is called a sampling frame (Särndal et al., 2003). In ideal situations, when sampling frames are available, "area frame" and "list frame" are the two most common types of frames that are used to design a household survey (Turner, 2003). These frames are developed usually by national statistical agencies, based on the information obtained from censuses. Therefore, these frames are supposed to be accurate and reliable at the date of conducting the censuses.

However, as time goes on, getting an up-to-date and reliable version of area frames and list frames, especially in regions where rapid changes are frequent, is very difficult. In these situations, alternative sampling frames (i.e., lists of residential postal addresses or lists of telephone numbers) and consequently different sampling approaches should be considered for household surveys.

Furthermore, sometimes the household surveys need to be conducted in a non-ideal situation where often the sampling frames are not available (e.g., designing a survey after a disaster or selecting a sample from poorly resourced settings). The absence of a well-defined sampling frame in non-ideal situations is a great challenge for survey statisticians and motivates them to use new technologies (Google Earth imagery, GIS, etc.) and new sampling schemes.

This chapter aims to investigate the feasibility of applying spatially balanced sampling methods for selecting spatial samples in the three different situations mentioned above (in the presence of an area frame or a list frame in ideal situations, in the presence of a list of residential postal addresses, and in non-ideal situations where there is lack of reliable sampling frame). The application of spatially balanced sampling methods for selecting samples in a two-stage cluster sampling design – which is a common household sampling design– will be studied in the next section. In that section, the precision of spatially balanced

sampling methods in selecting primary sampling units from an area frame in the first stage of a two-stage cluster sampling design will be compared with the precision of a probability proportional to size (PPS) without replacement sampling method. Then, for sampling in the second stage, the precision of the spatially balanced sampling methods will be compared with the precision of systematic sampling for selecting ultimate units from a list frame.

The third section of this chapter describes how spatially balanced sampling methods can be used to select sample households directly from a list of addresses of households. A modification of the BAS-Frame technique for conducting a spatial single-stage and a spatial two-stage cluster sample will be introduced.

Finally, the fourth section will explain the application of the BAS method for selecting samples in non-ideal situations.

6.2 Spatially Balanced Sampling Methods in Conducting a Two-Stage Cluster Sampling

6.2.1 Stage 1 – Selecting Sample Area Units in the Presence of an Area Frame

An area frame in a household survey is usually made up of geographical units of a country that are arranged hierarchically. This frame typically includes some features such as cities, districts, villages in rural regions and blocks in urban regions. Each feature in an area frame is assigned a unique code. Geographical units in an ideal area frame cover the entire area of the target population and each unit has well-defined boundaries. A list of meshblocks in Christchurch is an example of an area frame.

As explained in Chapter 2, area frames are usually used at the first stage of the sample selection process in household surveys. In classical household sampling techniques, sample area units are often selected from area frames using a PPS sampling method. However, this sampling method does not guarantee that the selected area units are well spread over the population.

With the increasing availability of geographical information (e.g., online maps, and satellite imagery), the sampling selection process in the first stage of a household sampling survey can be enhanced by employing spatially balanced sampling methods. In this subsection, the effect of using spatially balanced sampling methods on increasing the representativeness of area samples was investigated through conducting a simulation study

on Christchurch meshblocks. The available area frame of meshblocks contains a list of the meshblocks' unique codes as well as the geographical coordinates of the geometric centre of each meshblock and some attributes related to each meshblock. The geographical positions of the centre of the meshblocks are shown in Figure 5-14. In this thesis, the geometric centre of meshblocks were calculated using "sp" package in R (R Core Team, 2017).

In the simulation study, sample area units (meshblocks) were selected using five different sampling methods:

1) PPS systematic (PPS-SYS) sampling without replacement where meshblocks arranged by their unique codes,
2) PPS-SYS sampling without replacement where meshblocks arranged firstly by their longitude and then by their latitude,
3) Spatially balanced sampling methods (LPM, BAS-Frame with adding random points and GRTS) with unequal sample inclusion probability,
4) Balanced sampling (CUBE method), and
5) Conditional Poisson (CP) sampling.

PPS-SYS sampling selects samples by using systematic random sampling with probability proportional to size. In this method sampling units are selected at a fixed selection interval throughout the sampling frame after selection of a random start. The systematic selection interval is the ratio of the total size to the sample size (M/n). If the size of the i^{th} unit (M_i) is greater than the selection interval, the i^{th} unit might be selected more than once. To avoid any duplications in the sample, the size of the i^{th} unit (M_i) is required to be less than M/n.

Poisson sampling is a sampling design for selecting samples with unequal inclusion probabilities. For Poisson sampling, each unit is selected according to an independent Bernoulli trial. This will provide a random sample size. CP is a special case of Poisson sampling introduced by Hajek and Dupac (1981) where the condition of selecting a sample with a fixed sample size is added. To achieve a fixed sample size, it is possible to generate Poisson samples and accept the sample only if it has the required sample size. In this study, CP is used as a benchmark method to compare the unequal probability sampling designs.

As previously discussed in Chapter 5, for selecting samples with equal inclusion probability by means of BAS-Frame, the process of creating a primary frame can be done

with either adding points to or removing points from the population of interest randomly. The simulation studies conducted in Chapter 5 showed that the application of the BAS-Frame technique results in more spatially balanced samples when the random points are removed. Therefore, the BAS-Frame technique with removing random points was suggested for selecting samples with equal inclusion probability. However, for selecting samples with unequal inclusion probability, random points could not be removed randomly as the population units may not have the same probability of selection. In these cases, the primary frame is created by adding random points that have selection probability equal to zero.

For all of the sampling designs, the inclusion probability is set to be proportional to the total number of households in meshblocks. In other words, the inclusion probability of i^{th} meshblock was calculated by $\pi_i = \frac{G_i}{\sum_{i \in U} G_i}$, where G_i is total number of households in i^{th} meshblock. In this simulation study, a meshblock's total income is considered as the response variable.

For each sampling technique, after selecting 1000 samples, the average of ζ, $\hat{\mu}(\zeta) = \frac{1}{1000}\sum_{i=1}^{1000}\zeta_r$, was calculated as an index to compare the spatial balance among the different sampling designs.

The precision of the five sampling designs were compared to each other through calculating the simulated variance of the HT estimator for total income and then using Equation (6.1).

$$Deff_{complex,CP} = \widehat{\mathrm{V}}_{complex}(\widehat{\mathrm{T}}_{\mathrm{HT}}) \Big/ \widehat{\mathrm{V}}_{CP}(\widehat{\mathrm{T}}_{\mathrm{HT}}) \tag{6.1}$$

where $\hat{V}_{complex}(\hat{T}_{HT})$ and $\hat{V}_{CP}(\hat{T}_{HT})$ are the simulated variance of the HT estimator among 1000 samples selected by the complex designs (PPS-SYS, spatially balanced sampling methods, balanced sampling) and CP, respectively. Like other simulation studies in this thesis, the samples were selected at five different sampling fractions (1, 2, 3, 4 and 5%).

The results of the simulation study are shown in Table 6-1, Figure 6-1 and Figure 6-2.

Table 6-1 Achieved $\hat{\mu}(\zeta)$ and $Deff_{complex,CP}$ using 1000 samples for different sampling fractions.

Sampling fraction	Index	Sampling design						
		PPS_SYS	PPS_SYS (ordered)	LPM	BAS-Frame	GRTS	Cube	CPS
1%	$\hat{\mu}(\zeta)$	0.33	0.14	0.12	0.20	0.16	0.43	0.86
	$Deff$	0.33	0.30	0.22	0.29	0.34	0.30	1
2%	$\hat{\mu}(\zeta)$	0.41	0.19	0.11	0.19	0.15	0.40	0.53
	$Deff$	0.15	0.16	0.12	0.12	0.18	0.17	1
3%	$\hat{\mu}(\zeta)$	0.33	0.22	0.10	0.22	0.15	0.40	0.47
	$Deff$	0.14	0.10	0.09	0.13	0.11	0.10	1
4%	$\hat{\mu}(\zeta)$	0.37	0.21	0.10	0.21	0.14	0.38	0.43
	$Deff$	0.09	0.05	0.07	0.09	0.09	0.05	1
5%	$\hat{\mu}(\zeta)$	0.37	0.25	0.10	0.21	0.14	0.36	0.41
	$Deff$	0.07	0.05	0.06	0.07	0.07	0.04	1

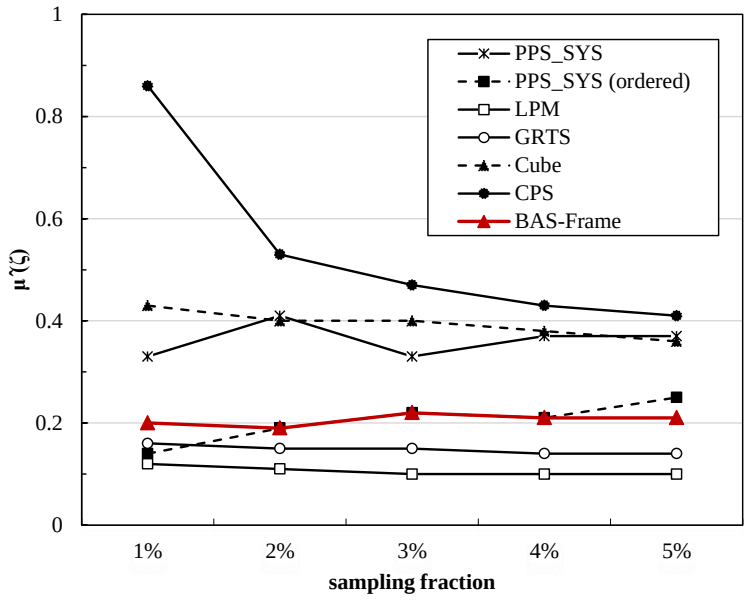


Figure 6-1 Achieved $\hat{\mu}(\zeta)$ for all evaluated methods and different sampling fractions.

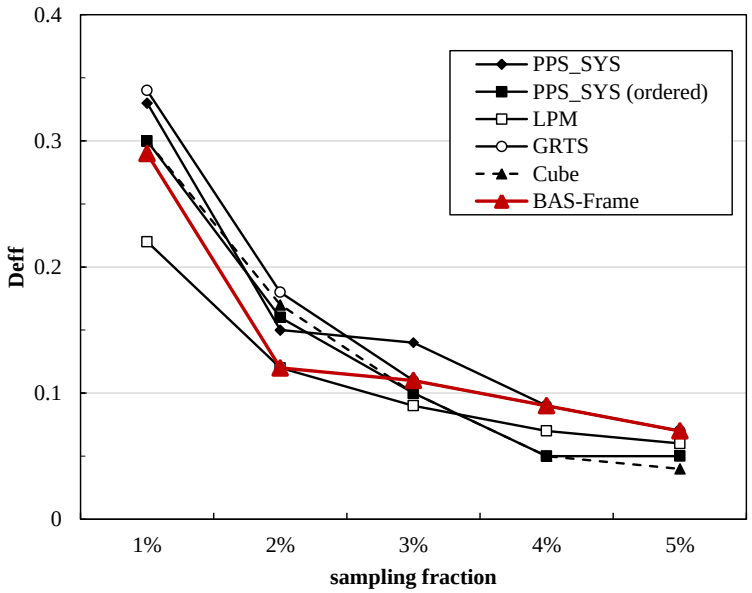


Figure 6-2 Estimated $Deff_{complex,CP}$ *for all evaluated methods and different sampling fractions.*

The results show that when the population units are assigned unequal inclusion probabilities, spatially balanced sampling methods (LPM, GRTS and BAS-Frame) spread sampling units over the population of interest more evenly than CPS and PPS-SYS. Among the spatially balanced sampling methods considered, LPM has the smallest value of $\hat{\mu}(\zeta)$, it shows that LPM generated more spatial balanced samples rather than the other methods. As Figure 6-1 shows, PPS-SYS can spread the sampling units over the population as well as spatially balanced sampling methods when the meshblocks are arranged firstly by their longitudes and then by their latitudes. In fact, ordering the population units according to their geographical locations means that the PPS-SYS sampling can select samples that are spread over the population as evenly as samples that are selected by spatially balanced sampling methods. Even though the PPS-SYS method is an easy-to-implement sampling method in household surveys, it may sometimes interact with a hidden periodic trait in a population. In fact, if there is a cyclical pattern in the population and the sampling interval coincides with the periodicity of the trait, the SYS method will no longer be random. Figure 6-1 also shows that the cube technique using the latitude and longitude of the centre of meshblocks as auxiliary variables did not work as well as spatially balanced sampling methods in spreading the sample meshblocks over the population.

Comparing the design effects of different sampling designs in Figure 6-2 illustrates that all the evaluated sampling designs had a smaller variance than the CPS. Although the smallest variances were associated with the spatially balanced sampling methods, there was no remarkable difference between results from the spatially and non-spatially balanced sampling methods. This showed that the implementation of the spatially balanced sampling methods in this example provided estimates which had similar precision to the estimates achieved by the non-spatially balanced sampling methods.

6.2.2 Stage 2 – Selecting Sample Households in the Presence of a List Frame

The list frame is another popular sampling frame that can be used for selecting sample households at the last stage of a household survey. As mentioned before, in most countries, censuses are major sources of generating a list frame of households. Therefore, the list frames may have discrepancies as the time interval between the household survey and the census is increased (Bycroft, 2011).

To address the lack of availability of a suitable list frame in household surveys, one may suggest constructing a frame through a field enumerating process. This practice is typically implemented before conducting the last stage of an area multistage sampling survey (Holzer et al., 1985; United Nations-Statistical Division, 2008; ICF, 2012) In a typical field enumeration process, field staff create a list of households (or dwelling units) in a small region of interest by starting from a predefined location and travelling within the region based on a specific rule. Figure 6-3, for instance, illustrates two different rules of listing paths, which were used in the Global Adult Tobacco Survey (Centers for Disease Control, 2010).

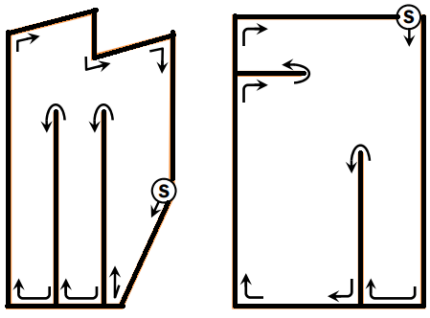


Figure 6-3 Two different rules of listing paths (Redrawn from Centers for Disease Control (2010)).

Once the household listing operation is completed, the created list frame is used for selecting the sample households. Usually sample households are selected by an equal

probability systematic sampling method. In fact, using the systematic sampling method at the last stage of a household survey aims to spread the sampling households over the region of the target population and prevent the selection of a collection of neighbouring households.

The use of GPS to record the geographical coordinates of households in the listing operation will provide some geographical visualization of the population units. Consequently, in addition to systematic sampling, other spatially balanced sampling methods can be used to select a well-spread sample.

In the previous chapters, the results of the simulation studies of applying spatially balanced sampling methods for selecting samples showed that the LPM and the BAS-Frame (with the random point removal option) are preferred to the other spatially balanced sampling methods in spreading out the sampling units over the population. The findings provided evidence that the LPM method and the BAS-Frame method can be used as alternatives to systematic sampling at the last stage of a sampling household survey.

Of note, however, is the inability of the LPM and BAS-Frame methods to select sample households at the time of generating a list frame. In fact, using a systematic sampling method at the last stage of a household sampling survey does mean that it is possible to extract sample households at the same time as providing a list of households. This is a practical advantage of systematic sampling when compared with LPM and BAS-Frame, because for these two methods the list frame need to be implemented after completing the field listing process.

Therefore, in practical cases where the household listing and sample selection process need to be done at the same time, systematic sampling might be a good solution. In cases where extracting the geographical coordinates of households without running a listing process is possible, using the LPM method is recommended as it is more effective in spreading the sample when the population size is fairly small.

6.3 Spatially Balanced Sampling Methods in the Presence of a List of Household Registry

In the first section of this chapter, the possibility of using spatially balanced sampling methods in the classical sampling designs currently used in household surveys has been studied. This section investigates how the spatially balanced sampling methods can be implemented with new forms of sampling frames.

The high cost of household listing and data collection in a face-to-face interviewing technique (an in-person survey), which has historically been used in household surveys, has motivated statisticians to use alternative sampling frame and/or interviewing techniques (Link et al., 2008). A telephone sampling survey based on random digit dialling (RDD) (Cooper, 1964) is an example of these alternative sampling methods. Population registers are also a new type of household sampling frame that have been used in European countries (Scherpenzeel et al., 2017). Population registers contain information about individuals who are living in a given country (Poulain et al., 2013). Furthermore, the growth in database technology has facilitated the use of computerised address datasets of residential locations. The Computerized Delivery Sequence (CDS) file of the United State Postal Service (USPS) is an example of a computerised address dataset in the United States that includes all delivery point addresses serviced by the USPS (United States Postal Service).

The existence of an updated address list of residential locations (i.e., CDS) enables statisticians to select sampling addresses directly. This sampling method is called address-based sampling (ABS) method (Link et al., 2008).

In ABS, the available address list is considered to be a sampling frame and addresses are selected randomly from it. Since the ABS usually provides access to households with more cost-effective instruments (such as mail, cell phones and/or internet facilities), there is no concern about the travelling costs associated with personal visit interviews. Due to this advantage, instead of using area-based sampling methods, sample households in an address-based sample can be selected directly using a spatially balanced sampling method. However, in cases where an address-based sample needs to be conducted through a face-to-face interview, spreading the sampling units over the population may increase the survey cost.

One might suggest adding census geographic entities (i.e., districts) to the list address and then extracting a sample using the conventional sampling methods. Or, as another solution, a modified version of BAS-Frame method will be introduced in this section, to select a sample from a list of registered households. Applying the modified version of BAS-Frame does not require adding any information to the list of the addresses.

6.3.1 Cluster BAS-Frame Method

Spatially balanced sampling methods aim to spread a sample over the population of interest. However, the selected sample may incur a high cost when responses need to be collected through a face-to-face interviewing technique.

In order to overcome this difficulty, the spatially balanced sampling methods can be modified into cluster sampling designs. This can be done by creating clusters of addresses at the first step and then selecting only some of the created clusters to sample.

The BAS-Frame technique can support the concept of cluster sampling by creating a primary frame (and consequently a regular frame) consisting of boxes with more than one unit. This technique is called the Cluster BAS-Frame.

Similar to the BAS-Frame method, Cluster BAS-Frame creates a primary frame by producing successive vertical and horizontal division of the population units. In the BAS-Frame method, the process of division is continued hierarchically until one unit in each box is achieved, whereas clusters in the Cluster BAS-Frame technique include more than one unit. In fact, in the Cluster BAS-Frame technique, the hierarchical division process stops earlier than in the BAS-Frame method. The achieved boxes in the final step of partitioning are called clusters. In the process of creating a primary frame, random points may be removed from or added to the population. This should to be done when the created boxes contain an odd number of units and they still need to be partitioned into smaller parts. In cases where the households in the population are assigned an equal inclusion probability, the primary frame is suggested to be created by removing random points. Removing points randomly from the population in the process of creating a primary frame provides equal sized clusters in terms of number of units. In cases that households in the population are assigned different inclusion probabilities (e.g., inclusion probabilities are proportional to the total number of adults in households), the primary frame may need to be created by adding random points. Note that, in this situation, the created clusters in the primary frame may have different sizes in terms of number of units. By introducing a suitable size variable and using the acceptance/rejection technique introduced by Robertson et al. (2013), the Cluster BAS-Frame technique is able to select unequal probability sample clusters.

The Cluster BAS-Frame tends to put nearby population units (i.e., households) in the same cluster and guarantees that the created clusters do not overlap each other. In addition, it ensures that the sample clusters are spread over the population.

Decreasing the survey cost is the main goal of the Cluster BAS-Frame technique, so this method does not provide a sample with the same spatial properties as the BAS-Frame method does in. Nearby households located in a same cluster are usually more similar to each other and consequently they provide similar information. Hence, for a fixed sample size, the estimates of the population characteristics achieved from the Cluster BAS-Frame can be less precise than estimates achieved from the BAS-Frame method. However, the trade-off between the spatial balance and the survey cost can be optimised by changing the number of units in the clusters. For a fixed sample size, as the number of units in the clusters is increased, the final selected households is less spatially balanced but less expensive. Losing precision in the estimates can be compensated for by selecting more clusters, although this comes with a higher cost. This general concept of the cluster sampling will be explored specifically for the Cluster BAS-Frame later in this section.

In a single stage Cluster BAS-Frame sampling, all households in the selected clusters are counted as sampling units, whereas in a two-stage Cluster BAS-Frame sampling, some households in the sample clusters are selected randomly at the second stage. Sampling selection in the second stage of a Cluster BAS-Frame sampling can be conducted through any probability sampling method as well as the BAS-Frame method. In a single stage Cluster BAS-Frame method, all units in the selected clusters are observed. Hence, the estimation techniques explained in Robertson et al. (2013) can be applied to this method by simply replacing "unit" with "cluster". The local mean variance estimator (Stevens, D. & Olsen, 2004) explained in Equation (2.18) can be used for variance estimation in the Cluster BAS-Frame technique. In a two-stage Cluster BAS-Frame method, the variance among the clusters can also be calculated using the local mean variance estimator (Stevens, D. & Olsen, 2004). The Cluster BAS-Frame method can also select samples from a population that is stratified either geographically or demographically. To generate a stratified Cluster BAS-Frame sample, the mutually exclusive strata are firstly defined, and then the Cluster BAS-Frame method is implemented in each stratum independently.

6.3.2 Application of the Cluster BAS-Frame Method

To demonstrate the potential of the Cluster BAS-Frame method and its suitability for application, the Cluster BAS-Frame method is used to select samples from an address list. This subsection compares the survey cost and precision of estimates when a spatial sample is selected by the Cluster BAS-Frame method rather than by using a conventional spatially balanced sampling method.

6.3.2.1 Generating an Artificial Dataset

The simulation study was carried out on an artificial address list of households, which has been generated based on some available information about meshblocks in Christchurch city. In addition to the geographical boundaries of meshblocks, the total number of one-storey and two-storey housing units within each meshblock were known. For simplicity, it was supposed that each storey of a housing unit is occupied by only one household.

To generate an address list of households, in the first step, sample points equal to the total number of housing units in each meshblock were generated within that meshblock's boundary randomly using the "sp" package in R. Then, the generated point locations were randomly labelled as a one-storey housing unit or two-storey housing unit. In the second step, point locations that have been dedicated to the two-storey housing units were doubled. This practice ensures that households that are living in the same housing unit have the same geographical location in the generated address list. The generated list contained 174,481 households.

Locations of the generated household addresses in two meshblocks in Christchurch are shown in Figure 6-4. Red points in Figure 6-4 indicate the location of housing units with two stories.

After generating the address list, a response variable, $income$, related to each household has been created according to the geographical locations of housing units using Equation (6.2):

$$income_i = (3(x_i + y_i) + \sin(6(x_i + y_i))) \qquad (6.2)$$

where x_i and y_i are the latitude and longitude of the i^{th} household. Income data usually follows a lognormal distribution (Darkwah et al., 2016). However, in this study Equation

(6.2) was used to generate random variables as it is in line with the assumption of this thesis that nearby households are more similar than household who are far away.

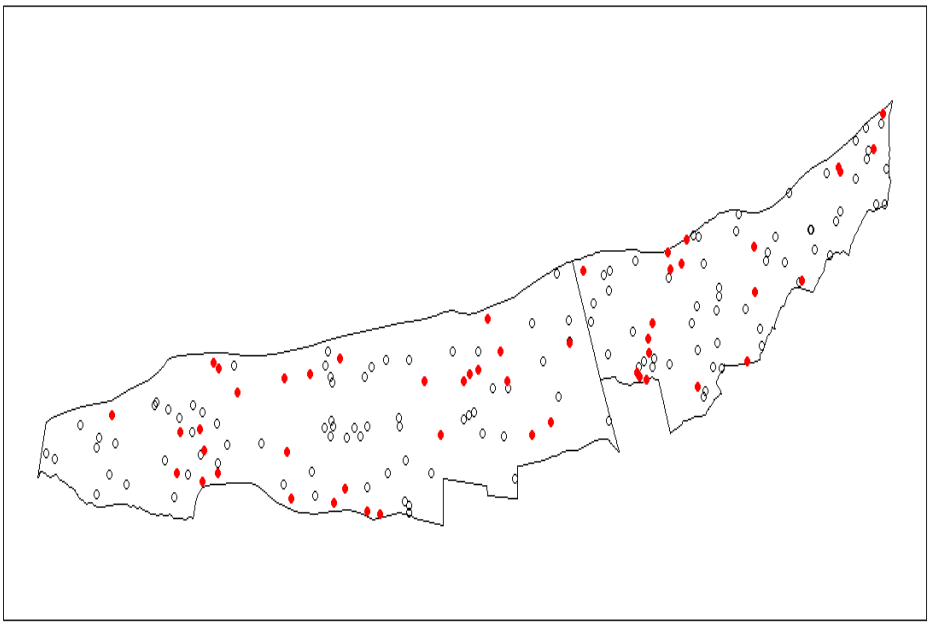


Figure 6-4 Locations of generated housing units in two meshblocks in Christchurch. Red points show the locations of two-storey housing units.

6.3.2.2 Sample Selection

The longitude and latitude of housing units in the generated address list provide spatial information that can be used as auxiliary information for selecting samples. As mentioned in the previous subsection, spatially balanced sampling methods and the Cluster BAS-Frame method are two potential sampling techniques that can select spatially balanced samples from this kind of frame. For comparing the applicability of the Cluster BAS-Frame method with the BAS-Frame method, 1000 samples were selected from the address list using these two methods. The LPM was removed from this simulation study as it takes too much computing time (about 7 minutes with a personal PC) to select a sample size of 20 households. The simulation study was carried out with three different sampling fractions (1, 2 and 3% of households) for selecting samples using the BAS-Frame method. To implement the Cluster BAS-Frame method, two options were considered, as follows:

a) the population was partitioned into boxes such that each box contains 85 households, then $n = 21, 41,$ and 61 boxes were selected as sample clusters.

b) the population was partitioned into boxes such that each box contains 42 households, then $n = 42, 83,$ and 124 boxes were selected as sample clusters.

Since population units possess equal inclusion probabilities, primary frames were created by removing points randomly.

Sample households in the BAS-Frame method were selected directly from the address list, while in the Cluster BAS-Frame method, clusters, all households in the selected clusters were considered as sampling units.

After selecting samples for each sampling scheme, the simulated variance of HT for estimating the $income$ average were calculated for the 1000 samples. The average of the smallest distance to visit all selected sampling households among all 1000 sample was also calculated using the travelling salesperson problem and "TSP" package in R. This study uses the default setting of function "solve_TSP()" in the package. Results of the simulation study for the three different sampling fractions are presented in Table 6-2.

Table 6-2 Simulated variance of HT estimator for estimating households' average income and the shortest distance (km) for visiting the selected sample among 1000 samples selected by the Cluster BAS-Frame and BAS-Frame method for a range of sampling fraction.

Sampling Fraction	Index	Sampling Scheme		
		BAS-Frame	Cluster BAS-Frame	
			cluster size =42	cluster size =85
1%	variance of HT(Cluster BAS-Frame) / variance of HT(BAS-Frame)	1	3.32	7.35
	smallest visiting distance	673,043	164,519	153,690
2%	variance of HT(Cluster BAS-Frame) / variance of HT(BAS-Frame)	1	3.98	6.97
	smallest visiting distance	939,603	301,305	218,697
3%	variance of HT(Cluster BAS-Frame) / variance of HT(BAS-Frame)	1	3.2	6.9
	smallest visiting distance	1,131,994	395,298	289,720

The results of the simulation study confirm the possibility of using the Cluster BAS-Frame method in selecting sample households from a register of households. As the results illustrate, for all the sampling fractions considered the shortest distance to visit the sample

households is significantly higher in the BAS-Frame method compared with the Cluster BAS-Frame method. This is because Cluster BAS-Frame selects a group of households that are located geographically near to each other. Table 6-2 shows that in a fixed sample size, by increasing the size of clusters, the shortest distance to visit all the sample households is decreased. The smallest visiting distances achieved confirm that using the Cluster BAS-Frame method can decrease the sampling cost.

Cluster BAS-Frame selects spatially balanced clusters; however, it may not be considered a spatially balanced sampling method in terms of selecting ultimate sampling units. Comparing the simulated variance of HT among the two sampling methods evaluated shows that the BAS-Frame technique provides more precise estimates than the Cluster BAS-Frame method.

According to the results achieved, employing the Cluster BAS-Frame is recommended in situations when the available sampling frame is a list of households in the population and there is an interest to decrease the sample costs in an in-person survey.

6.4 Application of Spatially Balanced Sampling Methods in Household Surveys in Non-ideal Situations

Up to this section, spatially balanced sampling methods were employed for selecting samples in ideal situations where either an area or list frame of the sampling units is available (i.e., selecting a sample of Christchurch meshblocks from a list including the geographical coordinates of the centre of Christchurch meshblocks). However, sometimes surveys need to be conducted in non-ideal conditions. Conducting a survey immediately after a disaster (e.g., an earthquake) or drawing a sample from war areas that have not had a census for a decade are two examples of non-ideal conditions. These situations are called non-ideal conditions as there is no reliable sampling frame for the population units and consequently the usual sampling methods, such as cluster or stratified sampling, might not be used as well as they could be, for gathering information.

Fortunately, recent advances in GIS and spatial tools have been shown to be helpful when conducting sampling surveys in non-ideal situations. For instance, Elangovan et al. (2016) used GIS/GPS-based grid-sampling method to study tuberculosis in Thiruneermalai, India. They overlaid a 30 × 30 metre small grid on the area of the population under study and selected 300 grid cells, using a simple random sampling method, as the sampling units. In

another study, on the migrants in Beijing, Landry and Shen (2005) designed a spatial sampling method to overcome the lack of complete coverage in traditional samples, in which the selections had been based on household lists. They firstly created a spatial grid of Beijing and then selected some cells randomly as sampling units. They demonstrated that their method reduced the coverage bias compared with traditional methods.

Most recently Thomson et al. (2017) have provided an R package to select samples from gridded population data. In their study, they used gridded population data as an alternative sample frame where census data was outdated or unreliable. The Gridded Population of the World (GPWv4), the Global Rural-Urban Mapping Project (GRUMP), LAndScan (LandScan Data Availability, 2017), WordPop (Stevens, F. et al., 2015), and Demobase (Azar et al., 2013) are examples of gridded populations that are available to freely download. Using the gridded population data, Galway et al. (2012) also designed a two-stage cluster sampling to study mortality in Iraq.

Kolbe et al. (2010) conducted a spatial sampling design to provide a rapid assessment of the population of the metropolitan area of Port-au-Prince soon after the earthquake of January 2010. They provided the geographic boundaries of the area at the first step and then selected a sample of GPS coordinates randomly within the metropolitan area. Other studies that have used spatial techniques to overcome the lack of sampling frames in households surveys can be found in Kondo et al. (2014), Haenssgen (2015), Siri et al. (2008), Shannon et al. (2012), Varona and Tabernilla (2013), Singh and Clark (2012).

6.4.1 Selecting a Spatially Balanced Sample From a Map Using the BAS Method

As discussed earlier, geographical information about the population under study is typically the only information that can be provided in a sampling frame in a non-ideal situation. In most situations, this frame is a map containing the geographic boundaries of different areas of the population. In this study, this kind of frame is called a map frame and might be provided using GIS technology. Figure 6-5 is an example of a map frame from a small part of Christchurch city.

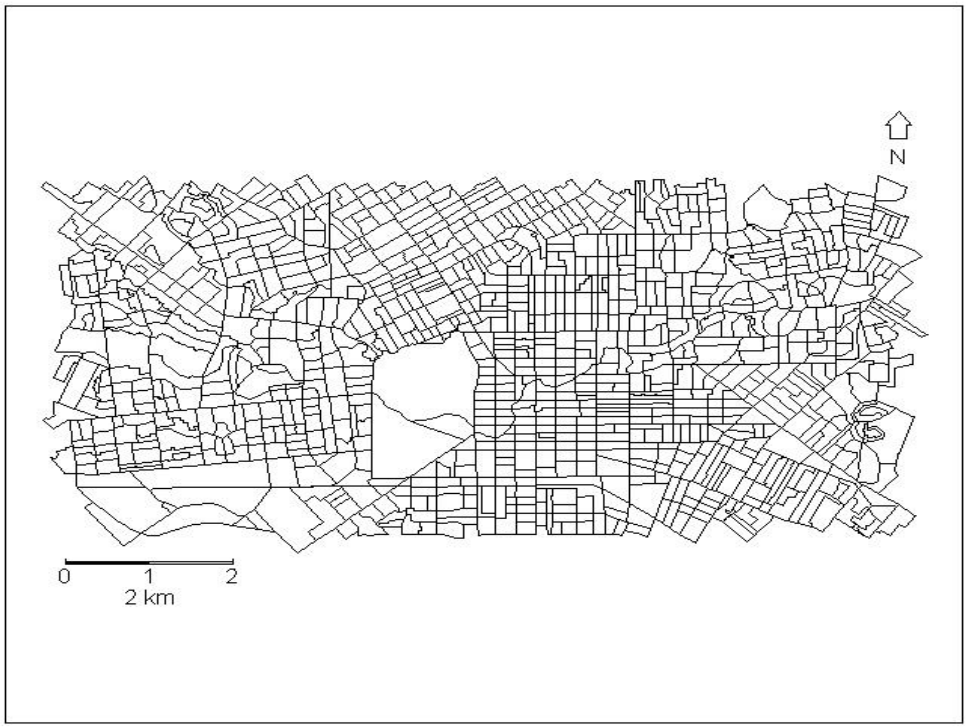


Figure 6-5 An example of an area frame shown on a map. In this map, the boundaries of the area units (meshblocks) are clear.

The geographical boundaries of the area units will provide a basis on which the centres of areas can be determined. This will allow the implementation of the available spatially balanced sampling methods (e.g., GRTS, LPM, BAS-Frame) for selecting spatially balanced samples from area units using the geographical locations of their centres. However, there are some cases that the centres of areas may not be located inside the boundaries. Figure 6-6 shows an example of this situation in which the centre of an irregular shaped area (Area 1) is located within the boundaries of the adjacent area (Area 2). In these situations, statisticians may prefer to select sample area units from a map frame using the boundaries of areas.

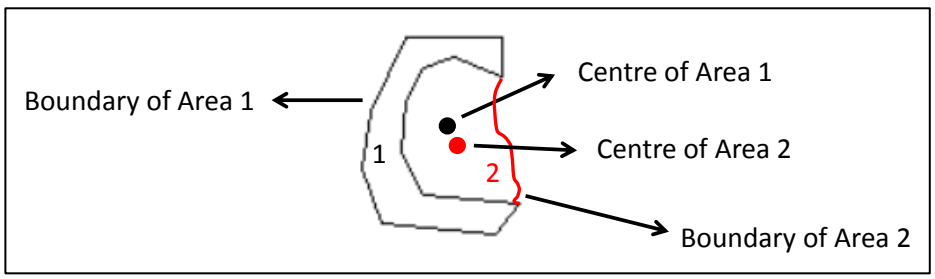


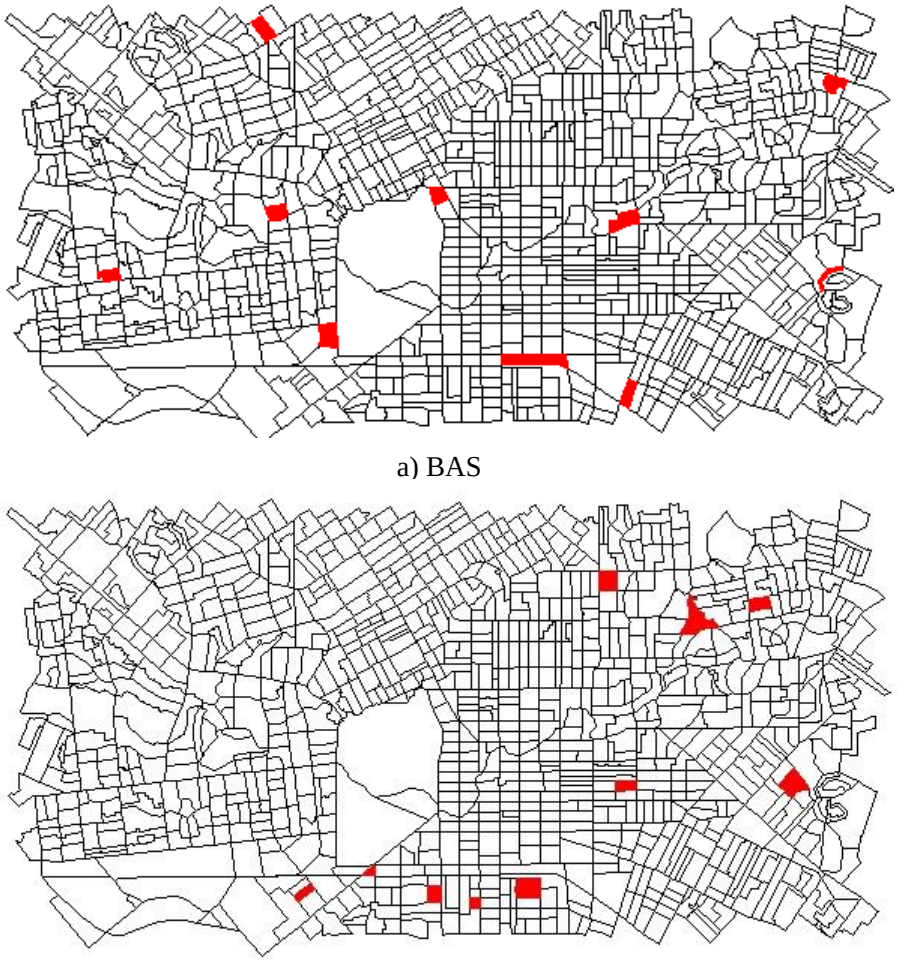
Figure 6-6 Illustration of a situation that centre of an irregular shaped area is not located within the boundaries.

BAS is one of the spatially balanced sampling methods that can be employed to select a spatially balanced sample of areas from a map frame. To carry out the BAS method, one can easily create a list of Halton points in two dimensions, as described in Chapter 3, and then take each Halton point in order. If the observed point falls inside the boundary of an area unit, that area unit is selected for the sample. If not, the Halton point is discarded, and the next point would be taken. This process is continued until the desired sample size has been reached. This works very well on a map that contains area units of the same size. However, map frames in reality are often made up of area units of different areas and shapes (such as those in Figure 5-14). Therefore, when BAS is used as the sampling method, the numbers of Halton points that fall in larger area units are likely to be more than the number of Halton points that fall in smaller area units.

Area units with different sizes can be given the same chance of being included in the sample by adding a dimension to the map (altering the two-dimensional map into a three-dimensional map) and applying an acceptance/rejection sampling technique. This additional dimension should be proportional to the inverse of the area of area units. In fact, adding an additional dimension to the map acts to adjust the effect of the areas of units on the sample selection process in such a way that the bigger units are given a smaller value on their third dimension and in contrast the smaller units are given a bigger value on their third dimension.

Implementation of the BAS method for selecting an equal probability sample from a map frame that includes area units of different sizes can be done by generating a sufficiently long list of a random-start Halton sequence in three dimensions. If the first two components of the first random-start Halton point falls in an area unit and the third component is smaller than the inverse of the area of the area unit, that area unit for the sample is selected; if not, the Halton point is discarded and the next random-start Halton point is checked. This process continues until the desired n sampling units have been selected.

Figure 6-7 illustrates a simple example of using the BAS method and SRS in selecting 10 meshblocks from a map frame of a small part of Christchurch city.



a) BAS

b) SRS

Figure 6-7 A sample of size 10 meshblocks selected by (a) BAS and (b) SRS from a map frame of a small part of Christchurch city.

As seen in Figure 6-7, the sample generated by BAS is visually more spread out over the population than the sampling units selected by SRS.

To study the applicability of the BAS method for selecting spatially balanced samples of areas based on their boundaries, a simulation study was carried out on a map of the meshblocks in Christchurch. This simulation study investigates whether a sample of areas which is selected based on the boundaries can be spatially spread to the same extent as a sample of areas which is selected based on the centre. In the first phase, the BAS method was used to select sample meshblocks according to their boundaries. In the second phase, sample meshblocks were selected using the geographical coordinates of their centres. In the latter

case, for selecting spatial sample, BAS-Frame with removing random points was used; SRS was used for selecting non-spatially balanced samples.

In addition to the evaluated sampling methods, a modification of SRS that can be used for selecting samples from a map was also considered in the simulation study. In the modification version of SRS, after selecting a random point within the boundary of the population, rejection/acceptance sampling was used to decide whether the corresponding meshblock to the selected point can be added to the sample. In fact, this modification of SRS works in the same way that the BAS on a map frame does, with this difference: that it generates sample points using pseudo-random numbers instead of quasi-random numbers. As was discussed in Chapter 3, in contrast to quasi-random numbers, pseudo-random numbers may fail to distribute the numbers evenly over the population.

The meshblocks which were used for the above-mentioned simulation study are illustrated in a density plot (Figure 6-8).

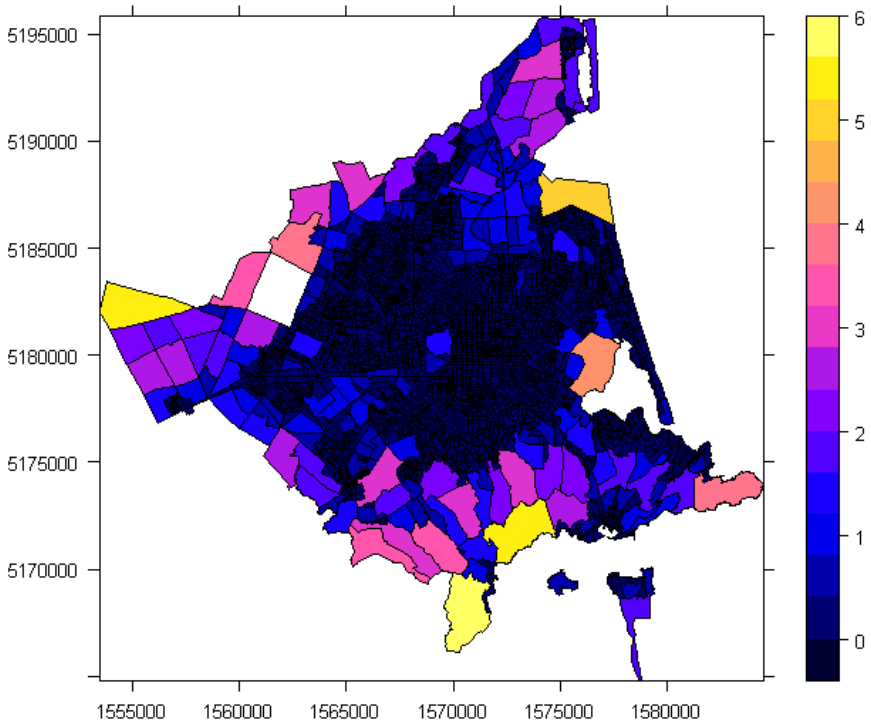


Figure 6-8 Spatial distribution of area of some meshblocks considered in the simulation study.

After selecting 1000 samples in five different sampling fractions (1, 2, 3, 4 and 5%), the average of ζ was calculated for the sampling methods considered. Note that ζ was

calculated using the geographical coordinates of centres of meshblocks. The results are reported in Table 6-3.

Table 6-3 The average of ζ among 1000 samples selected by four different sampling methods (i.e., BAS on a map frame, SRS on a map frame, BAS-Frame using centre of areas, and SRS) and for five different sampling fractions.

Sampling fraction	Sampling design			
	BAS from map	SRS from map	BAS-Frame	SRS
1%	0.19	0.37	0.14	0.38
2%	0.20	0.39	0.12	0.37
3%	0.21	0.40	0.12	0.39
4%	0.22	0.34	0.14	0.33
5%	0.21	0.33	0.13	0.37

As can be seen from Table 6-3, using the BAS method for selecting meshblocks from the map is superior to the modification version of SRS in terms of spreading the sample meshblocks over the population. Therefore, in cases that the selection of samples based on the boundaries of areas is desired, the use of the BAS method is suggested for selecting spatially balanced samples.

It was also found that the use of BAS-Frame (where the sample is selecting using the centres of the meshblocks) resulted in more spatially balanced samples compared with the situation that sample meshblocks were selected from the map using the BAS method. Hence, in surveys for which the centres of areas are considered in sample selection, to achieve more spatially balanced samples, it is suggested to employ the BAS-Frame.

6.5 Conclusions

Generally, a proper sampling design in a household survey is selected according to the characteristics of the available sampling frames. Area frame and list frame are the two main kinds of sampling frames in household surveys. The first section of this chapter used spatially balanced sampling methods for selecting sample units from these two types of frames. The results of the simulations performed in this part show that the current sampling methods applied in most household surveys can be substituted with spatially balanced sampling methods.

After introducing the new sampling frames, which are mostly used in developed countries, the application of spatially balanced sampling methods for selecting samples from these kinds of frames was investigated in the second section. Although employing spatially balanced sampling methods in the presence of the new format of sampling frames can provide more precise estimates, it may cause a high sampling cost. This drawback was addressed by modifying the BAS-Frame method. This modified version, the Cluster BAS-Frame method, decreases the survey cost by selecting units near to each other. It follows the same rationale as the BAS-method for putting the population units into clusters of more than one unit. It then uses the BAS method for selecting a spatial sample cluster as sampling units at the first stage of a multi-stage sampling process. The simulation study in this part showed that the shortest distance to visit all the selected sampling units can be halved when the samples were selected by the Cluster BAS-Frame method instead of the BAS-Frame method.

The application of spatially balanced sampling methods in a non-ideal situation where there exists no ordinary sampling frame was explained in the third section. After providing a map of the population of interest and defining the geographical centre of areas, the available spatially balanced sampling methods can be applied to select sample units. However, there are cases where statisticians prefer to select sample areas based on their geographical boundaries. Among the spatially balanced sampling methods, BAS can be used for selecting a sample directly from a map of area units that are defined only by their boundaries. This can be done by introducing an extra dimension relevant to the inverse of the area of each unit and then implementing an acceptance/rejection technique. A simulation study was conducted to compare the efficiency of the BAS method with the BAS-Frame method and a modified version of SRS in terms of selecting spatially balanced sample areas from a map. The results illustrated that the BAS method could generally select more spatially balanced samples compared to the modified version of SRS. In addition, it was found that the implementation of the BAS-Frame method on the basis of the centres of the areas provided more spatially balanced samples compared with the situation where sample areas were selected from the map using the BAS method.

6.6 References

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Chapter 7 *Spatially Balanced Sampling Methods and Some Features of Household Surveys*

7.1 Introduction

In the previous chapter, the application of spatially balanced sampling methods for selecting samples from different types of sampling frames in household surveys was investigated. In practice, household surveys often have specific requirements, for instance, the need to monitor an estimate of a parameter of interest over time, with demands for a particular survey design. This chapter describes how spatially balanced sampling methods can be used in these situations.

Constructing primary sampling units (PSUs) that are of pre-specified size and contain neighbouring units is one of these situations that will be discussed in the first section. The second section explains how spatially balanced sampling methods can be used in a longitudinal survey. Avoiding selection of the same units with multiple surveys is one challenge in household surveys that will be discussed in this section. Finally, the third section investigates how auxiliary variables can be used in designing a spatially balanced sampling method. A simulation study, based on real data, is used to investigate the efficiency of spatially balanced sampling with auxiliary variables.

7.2 Constructing PSUs in Household Surveys

Defining PSUs is one of the most important, and sometimes grueling processes, in designing a household sampling survey. Although natural geographical areas (such as meshblocks or counties) are considered to be practically the best candidates in terms of allocation of interviewers to areas and controlling the survey cost for PSUs in most household surveys, they often need to be modified or adjusted before being used as PSUs (Yansaneh, 2005). This modification is done to ensure that the selected PSUs include enough sampling units (Yansaneh, 2005). In fact, there is usually a requirement of having a pre-specified minimum number of secondary sampling units per PSUs.

Extremely large geographical units can usually be split into a number of smaller subunits, with one randomly selected as the PSU. This is called "segmentation". Another way to deal with extremely large geographical units that must be represented in the survey is to treat them as separate strata. In this situation, each large PSU is located in a separate stratum which is called a "certainty" or "self-representing" PSU (Kalton & Anderson, 1986). A self-representing PSU is in fact a stratum with only one member that is selected in the first stage of the sample selection process.

Undersized geographical units have sizes (e.g., number of their households) that are smaller than a pre-specified size and are usually combined with bigger ones to create PSUs which satisfy a pre-specified size. In the process of defining PSUs, dealing with undersized PSUs is generally more challenging than dealing with oversized PSUs. That is because combining undersized PSUs needs to be done prior to the selection of PSU. Whereas, the partitioning required for the oversized PSUs is done only when an oversized PSU has been selected. This section focuses on providing a simple method for combining adjacent undersized PSUs.

A procedure for combining PSUs during or after sample selection was first introduced by Kish (1965). After preparing a list of population units along with their size, the Kish method searches for the units which are either below the pre-specified size or immediately follow a unit below the pre-specified size. It then combines such units together. The list of population units can, for instance, be provided by ordering population units with respect to their longitude. Units with the same longitude are then ordered by their latitude. Given that units which are located geographically close to each other are listed near to each other on the list (e.g., when the serpentine ordering (Williams & Chromy, 1980) is used to sort units in a frame), this method tries to combine undersized nearby units with each other. The following example illustrates the procedure.

Example 7.1

Consider a population of 16 geographical units, labelled A, B, …, P. Let the size of each unit be defined by the number of households it contains. Here, the units are ordered firstly by their longitude and then by their latitude. Geographical units along with their sizes are shown in Table 7-1.

Table 7-1 Geographical units along with their sizes.

Unit ID	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Size	26	25	37	7	1	26	25	22	6	17	13	10	11	13	28	26

Suppose, in a household survey, PSUs need to be constructed from the geographical units in such a way that each of them contains at least 25 households while still being as small as possible. Thus, each geographical unit that has a size greater than 25 households can be defined as a single PSU, whereas geographical units with sizes smaller than 25 households are considered as undersized units. In order to make PSUs that satisfy the pre-specified size, the undersized units need to be combined with other geographical units. The Kish method for generating PSUs with desirable size follows these steps:

1. Define geographical units which either itself or its next following unit has less than 25 households. These units are marked by ✓ in Table 7-2.

Table 7-2 Geographical units which either itself or its next following unit has less than 25 households in the considered population.

Unit ID	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Size	26	25	37	7	1	26	25	22	6	17	13	10	11	13	28	26
			✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓		
			———				—	——		———		———				

2. Start from the last marked geographical unit and combine it with other geographical units, working backwards through the list. Once the preferred size (25 households) is reached, the combined geographical units are considered a single PSU. In this example the combined geographical units are illustrated in Table 7-2 with underlines.

Using the Kish method, 16 geographical units have been collapsed into 10 PSUs each of which has at least 25 households. These PSUs are: A (with 26 households), B (with 25 households), {C, D, E} (with 45 households), F (with 26 households), G (with 25 households), {H, I} (with 28 households), {J, K} (with 30 households), {L, M, N} (with 34 households), O (with 28 households) and P (with 26 households).

7.2.1 Using the BAS-Frame Technique for Combining Undersized Neighbouring Units

The Kish method combines PSUs according to their order in a list and there is no guarantee that the created PSUs would be constructed of nearby geographical units. Hence, the Kish method is not recommended for cases where there are a large number of undersized PSUs (Yansaneh, 2005) and PSUs need to contain nearby units.

Thomson et al. (2017) introduced a method for constructing PSUs when gridded population data are used as the sampling frame rather than census data. In their method, some cells (based on the sample size) are selected randomly from the gridded dataset in the first step as "PSU seed cells", and then the selected PSU seed cells will grow by adding neighboring cells one cell a time until a minimum PSU size is achieved. Each PSU seed cell will be expanded by randomly adding one of the nearest north, east, south, or west cells to the PSU. In this method, after selecting PSU seed cells, Voronoi polygons around each PSU seed cell are drawn, and the PSU growth is restricted inside the Voronoi polygons around each selected PSU seed cell. This ensures that the created PSUs do not overlap.

In this subsection a technique for combining undersized units in two-dimensional populations – defined by their geographical coordinates (latitude and longitude) – will be introduced. The method introduced by Thomson et al. (2017) only works on gridded population data, whereas the proposed method can work on all kind of datasets that contain geographical coordinates of units (i.e., census data and gridded population data). Another advantage of this method is that it provides a list of desirable sized PSUs that can be used as a sampling frame for a number of household surveys, not only a specific survey. Another difference between the proposed method and the Thomson et al. (2017) method is that there is no need to select PSUs seed cells or define Voronoi polygons.

The proposed technique is based on the rationale of the BAS-Frame method and should be implemented before employing the sample selection process. Similar to the BAS-Frame method, this technique provides a frame by partitioning the primary units (e.g., meshblocks) sequentially along their latitude and longitude. However, in this technique, the partitioning process is undertaken irrespective of the size of the primary units (e.g., number of households in each meshblock). In fact, the population is partitioned such that the creation of boxes smaller than a pre-specified size would be prevented. For this, the partition proposed in each

step (vertical or horizontal division) is accepted if the total size of secondary units (e.g., number of households) located in each created box is greater than the pre-specified size. The process of combining undersized primary units in the proposed method is as follows:

a) Determine the median of primary units along their first coordinate axis. This means the region of the population of interest is divided into two parts with the same count of primary units based on the first coordinate axis.
b) If the number of secondary units corresponding to the primary units which are located below (or above) the median is equal to or greater than the pre-specified size, the median split is accepted. In the case that the number of primary units is odd, before continuing the division process, an extra primary unit with size equal to zero is added to the box that is being split. In this technique, the primary units could not be removed randomly. That is because this technique needs to provide a frame that includes all the population units, and also to avoid changes of the size of the boxes which is likely to occur if the primary units are removed randomly.

The process is hierarchical: step (a) at the beginning targets the whole area of the population of the interest; however, in the repeat steps, it is applied within the created boxes. Steps (a) and (b) are repeated on each of the created boxes until the size of each box is greater than or equal to the pre-specified size.

To get an idea of how the proposed method can be implemented, the steps required for creating PSUs are illustrated through a simple example.

Example 7.2

Let Figure 7-1 illustrate the geographical position of units in the population described in Example 7.1. The size of each unit is shown inside the relevant brackets.

M(11)	N(13)	O(28)	P(26)
I(6)	J(17)	K(13)	L(10)
E(1)	F(26)	G(25)	H(22)
A(26)	B(25)	C(37)	D(7)

Figure 7-1 The geographical position of units in the population described in Example 7.1.

Like in Example 7.1, the units with less than 25 households are considered undersized and need to be combined with other units. The proposed method combines undersized units with their nearby units through the steps below:

Step 1 – the population units are temporarily split into two parts according to their first coordinate axis. The vertical temporary boxes achieved in this step are shown in Figure 7-2a. The dashed line in Figure 7-2a is used to show that these created boxes are still temporary. The total numbers of households in the created vertical temporary boxes are shown in Figure 7-2b.

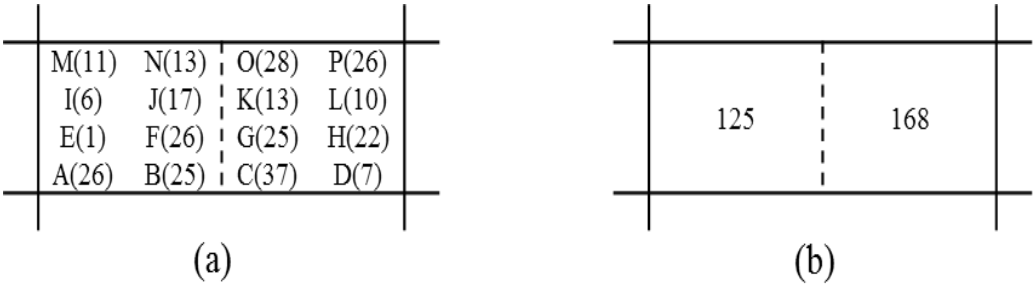


Figure 7-2 (a) Vertical temporary boxes achieved after completing the first step of the division, (b) total numbers of households in each created vertical temporary box.

The total sizes (total number of households) of the created vertical temporary boxes (125, 168 households) are greater than the pre-specified size (25 households), therefore the vertical division is accepted.

Step 2 – the units in each box are temporarily divided into two parts based on the second coordinate axis. The horizontal temporary boxes are separated from each other by dashed lines in Figure 7-3a. Total sizes of units in the created horizontal temporary boxes are shown in Figure 7-3b.

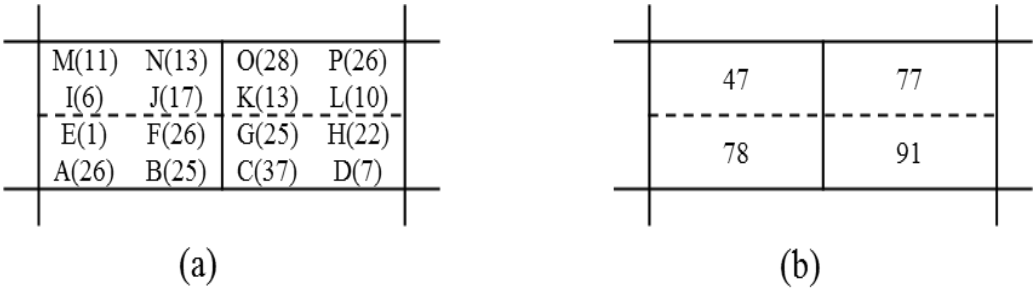


Figure 7-3 (a) Horizontal temporary boxes which are achieved after completing the second step of the division,(b) total numbers of households in each created horizontal temporary box.

Since the total sizes calculated in the horizontal temporary boxes created (78, 91, 47, 77 households) are greater than 25, the horizontal division is accepted.

Step 3 – units in each horizontal box created in the previous step are again temporarily divided into two parts based on the first coordinate axis. The created vertical temporary boxes in this step are depicted in Figure 7-4a. Sizes of the vertical temporary boxes created in this step are shown in Figure 7-4b.

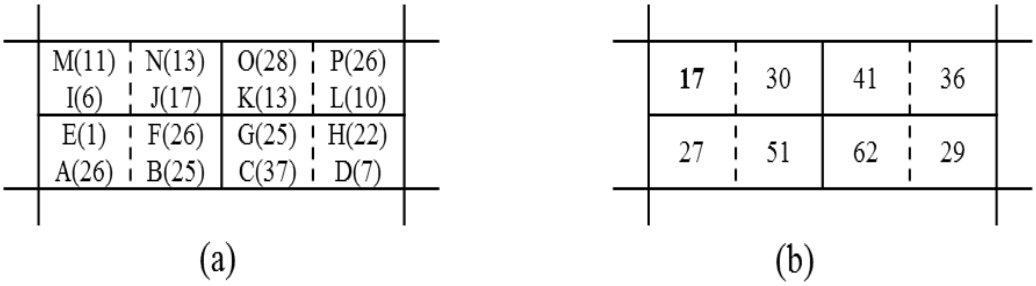


Figure 7-4 (a) Vertical temporary boxes achieved after completing the third step of the division, (b) total numbers of households in each created vertical temporary box in the third step.

The calculated total size related to the top left temporary box (shown in bold type in Figure 7-4b) is smaller than 25 households. Therefore the temporary created division could not be accepted in this stage. The final vertical boxes created in this step and their relevant sizes are shown in Figure 7-5a and Figure 7-5b, respectively.

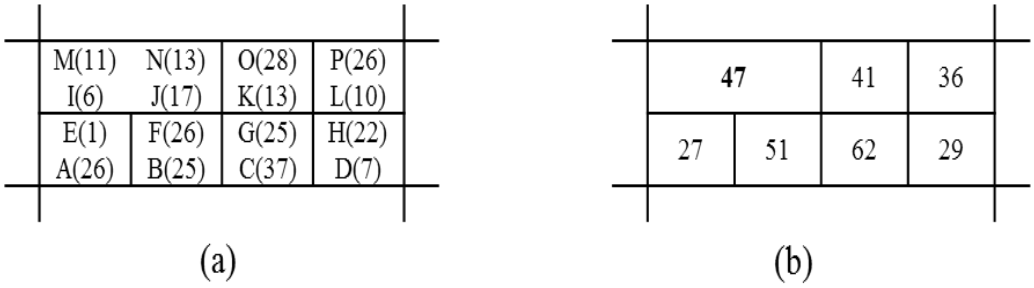


Figure 7-5 (a) Vertical permanent boxes achieved after completing the third step of the division, (b) total numbers of households in each created vertical permanent box in the third step.

After continuing the horizontal division processes for one more step, the pattern of the combined undersized units in Figure 7-6 would be achieved. The resulting boxes and their relevant sizes are illustrated in Figure 7-6a and Figure 7-6b, respectively.

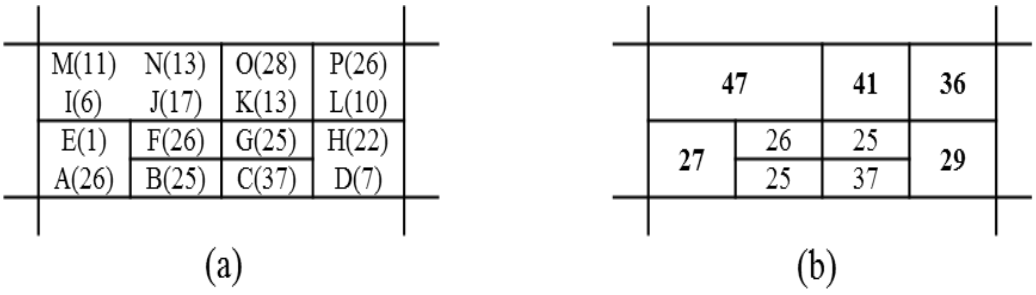


Figure 7-6 (a) final boxes after completing the division process, (b) total numbers of households in each created box after completing the division process.

As can be seen from Figure 7-6, all combined units have sizes greater than 25. Using the proposed method, 16 geographical units have been transformed into 9 PSUs with more than 25 households. These PSUs are: {A, E} (with 27 households), B (with 25 households), C (with 37 households), {D, H} (with 29 households), F (with 26 households), G (with 25 households), {I, J, M, N} (with 47 households), {K, O} (with 41 households) and {P, L} (with 36 households).

7.2.2 Application of the Proposed Technique on the Christchurch Meshblocks

To understand how the proposed technique performs in combining undersized units with their nearby units, the method was applied on the Christchurch meshblocks to create PSUs by combining small meshblocks with their nearby units. In this study, the number of households living in each meshblock was considered as the size of that meshblock.

As previously discussed, a method that combines the undersized meshblocks that are near to each other is more desirable in household surveys. In this subsection, the Kish method and the proposed technique were compared. The method introduced by Thomson et al. (2017) was not considered in this study, as its application is limited to gridded data. The comparison was based on the shortest distance between centres of meshblocks which constitute that PSU to understand which one is more successful in combining nearby meshblocks to form PSUs. The distance was calculated using the travelling salesman problem (TSP, Hahsler & Hornik, 2007). The goal of TSP is to find the shortest tour that visits each city in a given list and returns to the origin city (Hahsler & Hornik, 2007). To define the distance between meshblocks, Euclidean distances between centres of meshblocks were used. The geometric centre of meshblocks were calculated using "sp" package in R (R Core Team, 2017). The

shortest tour distances were also calculated using the default setting of function "solve_TSP()" in the package "TSP" in R.

After combining the meshblocks, P PSUs are created; and d_i $(i = 1, \dots P)$ is the shortest tour to visit centres of meshblocks which constitute the i^{th} PSU. In cases that PSUs consist of a single meshblock only, d_i is equal zero ($d_i = 0$). Once the tour distances were calculated for all created PSUs by using the default setting of function "solve_TSP()" in "TSP" package in R, the average distance required to visit meshblocks in the created PSUs ($\bar{d}$) was determined using Equation (7.1).

$$\bar{d} = \frac{1}{P}\sum_{i=1}^{P} d_i \tag{7.1}$$

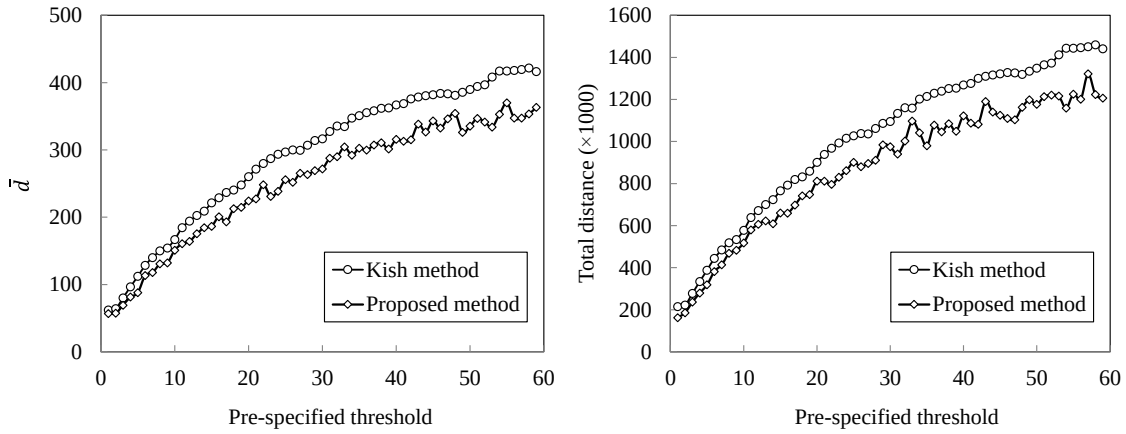
where

P: total number of created PSUs, and

d_i: shortest tour to visit meshblocks that constitute the i^{th} PSU.

For the purpose of this study, a range of sizes from 2 to 60 households was considered as pre-specified thresholds to form the desired PSUs. This range of household was considered on the basis of median of households in the Christchurch meshblocks. For each pre-specified threshold, $\bar{d}$ was considered as an index to compare the methods (Kish method and the proposed method).

For each pre-specified threshold, the proposed technique was repeated 1000 times. The average value of $\bar{d}$ associated with 1000 repetitions at each pre-specified threshold was then compared with the corresponding value obtained by the Kish method. Figure 7-7a shows the average distances ($\bar{d}$) for the PSUs determined using each of the methods for a range of pre-specified threshold levels. The total distance to visit all the created PSUs is also shown in Figure 7-7b.



(a) (b)

Figure 7-7 (a) Average distances ($\bar{d}$) calculated using both methods for a range of pre-specified PSU size thresholds varying from 2 to 60 households. (b) The total distance to visit all the created PSUs.

As can be seen from Figure 7-7a, for all pre-specified thresholds, the values of $\bar{d}$ calculated using the proposed method are smaller than their relevant values when the Kish method was used for forming PSUs with desirable values. This implies that the proposed method was more successful than the Kish method in combining the undersized meshblocks with other meshblocks located close to each other. Figure 7-7b also shows that for both methods, increasing the pre-specified threshold led to increases in the average of the distances. The study showed that the proposed method is promising for creating desirable sized PSUs in household surveys. Units (e.g., meshblocks) that constitute PSUs in this method are closer to each other than sampling units which constitute PSUs in Kish method. As such, the application of the proposed method will reduce the survey cost for visiting sampling units that are located in a same PSU.

7.3 Spatially Balanced Sampling Methods and Longitudinal Designs

Another requirement in some household surveys is to design the survey such that in addition of estimating parameters of interest at a fixed time (cross-sectional estimates), the changes in those parameters can be monitored on multiple occasions over a time period (longitudinal estimates). To meet this goal, rotation panel sampling which is a sampling technique in longitudinal surveys has become popular during recent decades (Steel & McLaren, 2009). For instance, Labor Force Surveys use a rotation panel sampling design in many countries (Steel, 1997).

In rotation panel sampling, a portion of sampling units is replaced with new sampling units on each occasion. A rotation panel sample is composed of equally sized sets of sampling units with a predetermined overlap between occasions. These sets, which are often a combination of some households, are called rotation groups. Typically, population units are systematically allocated to the rotation groups such that there is no overlap between rotation groups and selecting neighbors in the same rotation group is avoided (Hussmanns et al., 1990).

In this section, there is an interest in investigating whether spatially balanced sampling methods can be used for constructing the rotation groups in rotation sampling designs for household surveys.

Among spatially balanced sampling methods that have been referred to throughout this thesis, GRTS and BAS offer the ability to add more units to the current selected sample without losing spatial balance (Stevens, D. & Olsen, 2004; Robertson et al., 2013). Their studies showed that after selecting a spatially balanced sample of size n using GRTS or BAS, the size of the sample can be extended to $n + 1$ or more while still maintaining the spatial balance. Based on this characteristic, these two methods have a potential to be used for selecting samples in longitudinal surveys (van Dam-Bates et al., 2018). In other words, after selecting spatially balanced sampling units for the first rotation group, the new sampling units can be added to form the next rotation groups. As such, sampling units are not only spatially balanced in their rotation groups, but also their aggregations over all rotation groups provide a spatially balanced sample.

Similar to GRTS and BAS, it is expected that BAS-Frame allows for adding new sampling units to the selected sample when the sampling units are selected from a finite population. Here, this intuition was tested through conducting a simulation study. The simulation study was also used to compare the spatial balance in BAS-Frame with GRTS when extra units were added to the sample. SRS was considered as a benchmark during the simulation study to determine how well these methods (i.e., BAS-Frame and GRTS) can create spatially balanced samples.

In the simulation study, 1000 artificial finite populations, each consisting of 1025 discrete units with irregular positions were generated. In each population, units were generated randomly over a 10m by 10m square. Synthetic populations were generated 1000

times to ensure that the results are reliable enough to represent the generated populations. The size of the populations was set to 1025 units ($2^{10}+1$) to represent a worst case scenario that needs to add extra units with zero inclusion probabilities.

From each population generated in this study (of size 1025 units), a sample of size $n=2$ units (the smallest sample size) was initially selected by each of the three sampling schemes listed above. New units were then added to the sample one by one over time following reverse hierarchical order for GRTS (Stevens, D. & Olsen, 2004) and Halton points sequence for BAS-Frame. The process of adding sampling units was continued until a sampling fraction equal to 50% of the population (512 sampling units) was achieved. The new units can be added to the sample using different schemes (e.g., 5 by 5, 10 by 10, etc.); however, in this study new units were added to the sample one by one. This will provide a basis to cover the other schemes of adding different numbers of new units to the sample.

For each generated population, and in each step of adding a new unit to the sample, the mean square error of sum of the inclusion probabilities of units in Voronoi polygons, ζ_{in} ($i=1,\dots,100$; $n=1,\dots,512$) explained in Equation (2.22), were calculated as a measure of spatial balance. Then, the average of ζ_{in} among 1000 generated populations ($\bar{\zeta}_n$) was calculated.

The ratio of $\bar{\zeta}_n$ for GRTS and BAS-Frame when compared to SRS and each other are plotted in Figure 7-8.

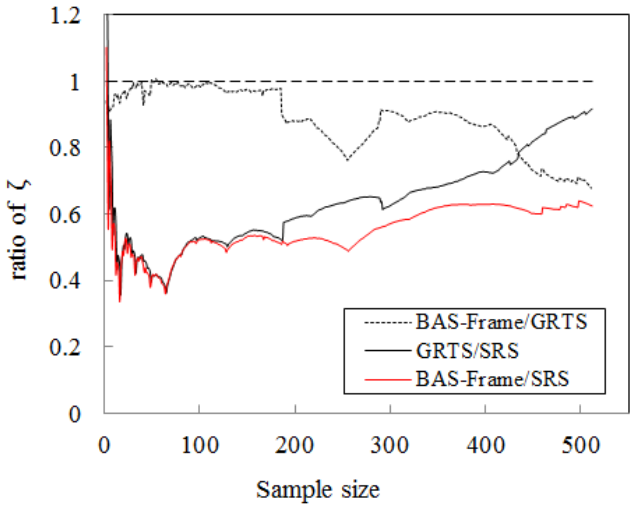


Figure 7-8 The ratio of $\bar{\zeta}_n$ for GRTS, BAS-Frame when compared to SRS and each other for a situation when sampling units are added to the sample one by one over a period of time.

Figure 7-8 shows, for all of the sample sizes considered in this study, the ratio of $\bar{\zeta}_n$ for both the BAS-Frame technique and GRTS was less than 1 when compared with SRS. The result associated with GRTS is in line with the previous studies (Stevens, D. & Olsen, 2004). The study also showed that BAS-Frame created a more spatially balanced sample than SRS when new sampling units were added to the sample. Although both techniques provided more spatially balanced sample compared to SRS, the ratio of $\bar{\zeta}_n$ for GRTS is greater than ratio of $\bar{\zeta}_n$ for BAS-Frame for all sample sizes. This means that the use of the BAS-Frame in adding new units to the sample results in more spatially balanced samples.

Based on the results derived from the simulation study, it could be concluded that the BAS-Frame method can be also employed in designing a rotation panel sample. This is because the BAS-Frame can add new sampling units to the sample such that the cumulative set of selected samples over the survey period is spatially balanced.

To implement BAS-Frame for creating rotation groups, after creating a long list of Halton sequence, sampling units (based on the rotation group's size) are selected (by the BAS-Frame method) to form the first rotation group. Subsequently, new sampling units are added to the sample to form the second rotation group. The process of adding new sampling units to the sample is continued to create all the rotation groups. In this process, rotation groups are created by tracing sequential points in the Halton sequence. Note that, Halton points associated with the sampling units selected in the previous rotation groups are no longer considered for the newly formed rotation groups.

Assuming that in a longitudinal sampling survey design, the 100 dwellings are required to be allocated into 20 rotation groups, the first 5 dwellings selected by BAS-Frame are considered as rotation group 1, the next 5 selected dwellings are considered as rotation group 2 and so on, and finally the last 5 selected dwellings are considered as rotation group 20. Note that, for each rotation group, sampling dwellings are selected by continuing from the last-used Halton point in the previous rotation group. Figure 7-9 illustrates the dwellings allocated into 20 different rotation groups in a population consisted of 100 randomly generated dwellings. Dwellings in the same rotation groups are shown in the same colour.

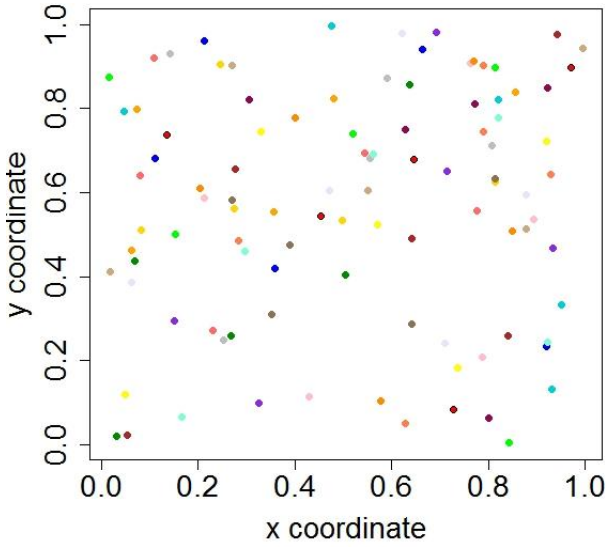


Figure 7-9 Sample dwellings allocated into 20 different rotation groups using the BAS-Frame technique. Dwellings with same colour are in the same rotation group.

After creating rotation groups, some of them are visited on each occasion according to a pattern which is called "rotation pattern" (Steel & McLaren, 2008, 2009). An example of a rotation pattern which is conducted quarterly for three successive years is shown in Figure 7-10. In this design, a sample of households is divided into 8 rotation groups. Each rotation group is interviewed for 8 successive quarters before leaving the sampling process. According to this rotation pattern, a new rotation group is entered to the sample for the first time in each quarter.

		Rotation Group																		
Year 1	Quarters	1	A_8	B_7	C_6	D_5	E_4	F_3	G_2	H_1										
		2		B_8	C_7	D_6	E_5	F_4	G_3	H_2	I_1									
		3			C_8	D_7	E_6	F_5	G_4	H_3	I_2	J_1								
		4				D_8	E_7	F_6	G_5	H_4	I_3	J_2	K_1							
Year 2	Quarters	1					E_8	F_7	G_6	H_5	I_4	J_3	K_2	L_1						
		2						F_8	G_7	H_6	I_5	J_4	K_3	L_2	M_1					
		3							G_8	H_7	I_6	J_5	K_4	L_3	M_2	N_1				
		4								H_8	I_7	J_6	K_5	L_4	M_3	N_2	O_1			
Year 3	Quarters	1									I_8	J_7	K_6	L_5	M_4	N_3	O_2	P_1		
		2										J_8	K_7	L_6	M_5	N_4	O_3	P_2	Q_1	
		3											K_8	L_7	M_6	N_5	O_4	P_3	Q_2	R_1
		4												L_8	M_7	N_6	O_5	P_4	Q_3	R_2

Figure 7-10 An example of a rotation pattern which is conducted quarterly for three successive years. Rotation groups are defined by alphabetic characters. The number of appearing of a rotation group in the sample is defined by its subscript: for example K_3 means that rotation group K is revisited for the third time. Rotation groups that are entered to the sample for the first time are shown in grey.

To investigate how well BAS-Frame performs for creating rotation groups compared to the conventional method (where dwellings are allocated into the rotation groups systematically), a simulation study was conducted on a population consisted of 100 randomly generated dwellings. It is worth mentioning that, the application of BAS-Frame method for selecting spatially balanced samples in comparison with systematic sampling method was previously discussed in Chapter 2. However, this section intends to compare the application of these methods for creating rotation groups in a longitudinal survey.

As discussed earlier, while the BAS-Frame technique allocates dwelling into the rotation groups based on the Halton sequence, in the systematic sampling method the allocation is based on a periodic interval. For creating rotation groups by using BAS-Frame, the dwellings in this case study were allocated into 20 rotation groups as explained above. In this example, in the process of creating the primary frame, random points with zero inclusion probability were added to the population. The addition of random points was preferred because it keeps all population units in the process of allocating them to the rotation groups.

In contrast, where dwellings were allocated systematically into the rotation groups, the dwellings were sorted according to their geographical coordinates and tagged from 1 to 100. A random dwelling amongst the first 20 dwellings was selected and allocated to the first rotation group. The next 19 successive dwellings were allocated into the other 19 rotation groups, one to one correspondingly. Assuming the tag of the selected dwelling is r, the dwellings $20 + r, 40 + r, 60 + r$ and $80 + r$ were also allocated to the first rotation group. The dwellings $21 + r, 41 + r, 61 + r$ and $81 + r$ were also allocated to the second rotation group. This process was repeated to allocate all remaining dwellings into the 20 rotation groups. The process of generating rotation groups was repeated 1000 times.

In this study, the rotation groups were visited according to the rotation pattern shown in Figure 7-10. The spatial balance of the selected sampling units in each quarter, which are measured by calculating the mean square error of inclusion probabilities in Voronoi polygons, explained in Equation (2.22), was calculated. The result of the simulation study is shown in Figure 7-11. In this figure "Y" denotes a year and "Q" denotes a quarter.

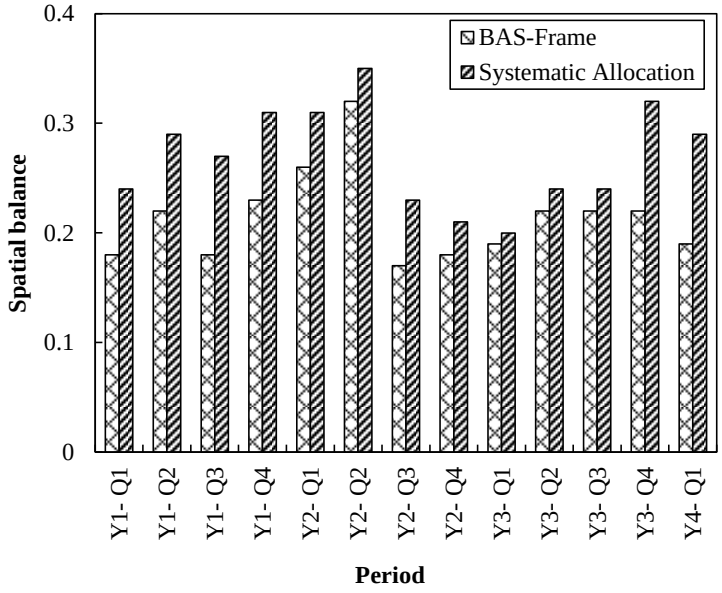


Figure 7-11 Spatial balance of the selected sampling units in each period.

In all periods considered in this study, the BAS-Frame created samples that were more spatially balanced compared with the dwellings systematically allocated to the rotation groups. This shows that BAS-Frame is a suitable alternative method for creating rotation groups in household longitudinal surveys.

In cases where the dwellings would be allocated into the rotation groups systematically, all the rotation groups would be created at the same time and before the sample collection has taken place. In contrast, the BAS-Frame method can select a new rotation group at the time of its application to the sample. This characteristic of the BAS-Frame method would increase its applications in creating the rotation groups in a longitudinal survey.

This study intends to use the BAS-Frame method to provide rotation groups that are spatially balanced and do not overlap each other. However, no estimators for estimating the parameters of interest (e.g. mean or total) were developed for the present purposes. Thus, there would be a need to expand the study in future work in an attempt to provide appropriate estimators for the parameters of interest.

7.3.1 Overlap Control between Different Household Surveys

National Statistical Agencies usually run a number of household sampling surveys at roughly the same time period. This means that it is possible to select a household in multiple surveys,

which will increase the undue respondent burden for that household. To reduce this burden, it is usually desirable to avoid selecting the same unit for more than one survey, while ensuring the units have their probabilities of selection for the survey to represent all of the population. Various procedures have been developed to minimize overlap with later surveys. A list of these procedures can be found in Ernst (1996, 1999), Chowdhury et al. (2000) and Lu (2012). In these procedures, the inclusion probability for each population unit is conditional on some aspect of its past usage to minimize the selection of units that have been selected before.

It was discussed that (Section 7.3) the overlap between rotation groups can be controlled by using the BAS-Frame method through discarding Halton points associated with the sampling units selected in several rotation groups. Discarding such repeated Halton points would result in selection of dependent samples. Such dependency is not desirable when it comes to selecting independent samples amongst different surveys. As such, repeated Halton points should not be discarded in the case of selecting independent samples. Therefore, the surveys might overlap each other.

However, as the size of the population increases, the boxes in the BAS-Frame get smaller. BAS points are spread evenly over the unit square, so when the BAS-Frame boxes are small, it is unlikely that multiple BAS points are selected in the same box.

To show the advantage of using BAS-Frame in avoiding selecting same sampling units for different surveys, a simulation study was conducted on the Christchurch meshblocks dataset which contains 2684 meshblocks. In the simulation study, it was assumed that three successive surveys (S1, S2 and S3) with three different sampling fractions (7%, 9% and 10%, respectively) need to be implemented independently on Christchurch meshblocks. For each survey, 1000 samples were selected using LPM, BAS-Frame and SRS method. After completing the sample selection, the average number of meshblocks that repeated in successive surveys was calculated. In both SRS and LPM, there was an average 4% overlap between samples of successive surveys together, whereas using the BAS-Frame created an average less than 1% overlap between successive surveys.

The results confirm the ability of the BAS-Frame method in conducting different household surveys such that sampling units do not overlap each other. BAS-Frame provides independent samples without making any change in the population units' inclusion

probabilities. This advantage of BAS-Frame highlights its potential application in providing official statistics.

7.4 Spatially Balanced Sampling Methods and Availability of Auxiliary Information in the Design Stage

Auxiliary information plays an important role in designing a sample for household surveys. In cases where only one auxiliary variable, which is correlated with the response variable, is available, it may be preferable to apply an unequal probability sampling method (i.e., PPS sampling method) to select more representative sample. In this situation the auxiliary variable is used as a measure of size of the population units. The application of spatially balanced sampling methods for selecting unequal probability samples in the presence of one available numerical auxiliary variable was shown in Section 6.2.1.

In cases with few qualitative auxiliary variables, these variables might be used in stratifying the population into some homogenous strata and applying a stratified sampling method to decrease the variance of population estimates. For instance, a stratified spatially balanced sample may be obtained in the simplest way by taking a spatially balanced sample in each stratum of the population separately. However, when there are many auxiliary variables, the stratified sampling may become more complicated in terms of finding the optimum number of strata and defining the strata boundaries. In these situations, instead of stratifying the population, it could be useful to extend the rationale of the spatially balanced sampling methods to spread the sample in the space of the auxiliary variables. In fact, it would be of more interest to select a well-spread sample, not only over the geographical region of the target population but also in the space of the auxiliary variables at the same time. LPMs and BAS are two popular spatially balanced sampling methods that can select samples from more than two-dimensions. As mentioned in Chapter 5, BAS-Frame can also select spatially balanced samples from more than two dimensions. This subsection investigates the efficiency of LPMs and BAS-Frame in spreading the sample in the space of available auxiliary variables in household surveys.

7.4.1 The Principles of LPMs and BAS-Frame in Spreading the Samples Over the Space of Auxiliary Variables

In a general format, the LPM methods select a sample by calculating distances (i.e., Euclidean distance) between population units. In the presence of auxiliary variables, further to

considering the geographical distances, the distances according to each auxiliary variable need to be calculated in order to identify close units in terms of that auxiliary information.

Assume for each unit in the population, there are m available auxiliary variables, where $\{1, \dots, k\}$ and $\{k+1, \dots, m\}$ correspond to the quantitative variables and qualitative variables, respectively. Grafström and Schelin (2014) calculated the distance between unit i and j among all the auxiliary variables by:

$$d(i,j) = \sqrt{\sum_{p=1}^{k} (x_{ip} - x_{jp})^2} + \sum_{p=k+1}^{m} I_p \qquad (7.2)$$

$$I_p = \begin{cases} 0 & x_{ip} = x_{jp} \\ 1 & x_{ip} \neq x_{jp} \end{cases}$$

where x_{ip} is the standardized value of the p^{th}auxiliary variable for unit i. Here, a standardized value is achieved by subtracting the minimum of the observations then dividing by their range. A weight matrix can be further included in the Equation (7.2) to account for the contribution of each auxiliary variable in defining distances between units. The total distance between unit i and j is obtained by adding $d(i,j)$ to the geographical distance between these units. In this thesis, the geographical distance between two units is defined by calculating Euclidean distance between their geographical coordinates. After calculating the total distance for all pairs of units in the population, a sample is obtained by applying the usual algorithm of the LPMs methods.

In addition to LPM, the BAS-Frame method is able to select a spatially balanced sample in the presence of the auxiliary variables. As presented in Robertson et al. (2013), being able to select a spatially balanced sample from a space of more than two dimensions is one of the advantages of the BAS method. For this, the latitude and longitude of the population units are taken as the first two dimensions and the m available auxiliary variables are taken as extra dimensions. To spread a sample in a m-dimensional space using the BAS-Frame method, the partitioning process should be carried out in all dimensions. The region of the population of interest is initially split on the basis of the geographical coordinates (i.e., longitude and latitude) of the units. The created boxes are then divided into two parts along the third coordinate axis (i.e., the first auxiliary variable). The partitioning process of the boxes is continued until all auxiliary variables are taken into account. Halton points are subsequently

generated in $(2+m)$ dimensions (two geographical dimensions along with m auxiliary variables). A unit is selected as a sampling unit if its corresponding box in the primary frame includes the generated Halton points in all dimensions. Note that, partitioning process may not be directly applied for the categorical auxiliary variables as they need to be firstly represented by numerical variables with jittered values. As such, in this study the BAS-Frame method has not been employed for spreading sampling units over the space of categorical auxiliary variables.

To investigate the possibility of using BAS-Frame in selecting a representative sample in the presence of auxiliary variables and compare it with LPM, a simulation study was performed on the Baltimore data set (Dubin, 1992). This dataset contains the selling price as well as other attributes related to 211 housing units. In this simulation study the "selling price in thousands of dollars (Price)" was considered as the response variable. In addition to geographical coordinates related to each house, three variables "Number of rooms (Nrooms)", "Age of dwelling, in years (Age)" and "Lot size, in hundreds of square feet (Lotsz)" were also considered as auxiliary variables.

A total of 1000 samples of sizes 10, 15, 20 and 25 out of 211 were selected by LPM and BAS-Frame. SRS was also considered in order to make a comparison between different designs. In this example, population units were assigned an equal probability of selection. As such, the primary frame required in the BAS-Frame method was created by removing points randomly from the population (as discussed in Chapter 5, when samples are selected by equal probability of selection, removing random points during the partitioning process results in more spatially balanced samples compare to a situation that random points are added to the population). By using LPM and BAS-Frame, we aim to spread the sampling units not only over the geographical region of the population of houses, but also over the space created by the three auxiliary variables to ensure that each considered auxiliary variable will be represented in the sample.

Similarly to other simulation studies implemented throughout this thesis, after defining Voronoi polygons related to each sampling units, the ζ explained in Equation (2.22) was used as an index for measuring how well spread the selected samples were. But, here, in addition to geographical distance between sampling units, the auxiliary distances between sampling units were considered for defining each Voronoi polygon. In fact, the ζ was calculated in five dimensions (two geographical dimensions and three auxiliary variables). Here, the "sb"

function available in "Balanced Sampling" package in R was used for calculating ζ. After selecting the 1000 samples, the average of spatial balance, ζ , was calculated for each sampling method. The results of the simulation study are reported in Table 7-3.

Table 7-3 The average of ζ among 1000 iterations for BAS-Frame and LPM in comparison with the relevant value for SRS.

Sample size	Design	
	BAS-Frame / SRS	LPM / SRS
10	0.51	0.49
15	0.54	0.49
20	0.56	0.49
25	0.63	0.50

As can be seen from Table 7-3, the ratio of ζ for both LPM and BAS-Frame in comparison to SRS is less than 1. This shows that these two methods spread samples more evenly over the region of population than SRS. To find out how representative the selected samples are, the distribution of sample means for each auxiliary variable was compared with its population distribution. The distribution of the sample mean of the auxiliary variables based on 1000 samples of size 10 for different sampling methods are presented in Figure 7-12. The true average value of each auxiliary variable in the population (5.2 for Nrooms, 30.1 for Age, and 72.3 for Lotsz) is also defined by vertical dash lines in its relevant distribution. Figure 7-12 shows that the sampling distributions obtained by all the three methods encompass the true values of parameters in the population.

The variance of the total estimation of the target variable (Price) and the other auxiliary variables (Nroom, Age and Lotsz) was also simulated using the simulated variance estimator in Equation (5.3). The simulated variances of the variables of interest for LPM and BAS-Frame in relation to SRS for four different sample sizes are shown in Table 7-4.

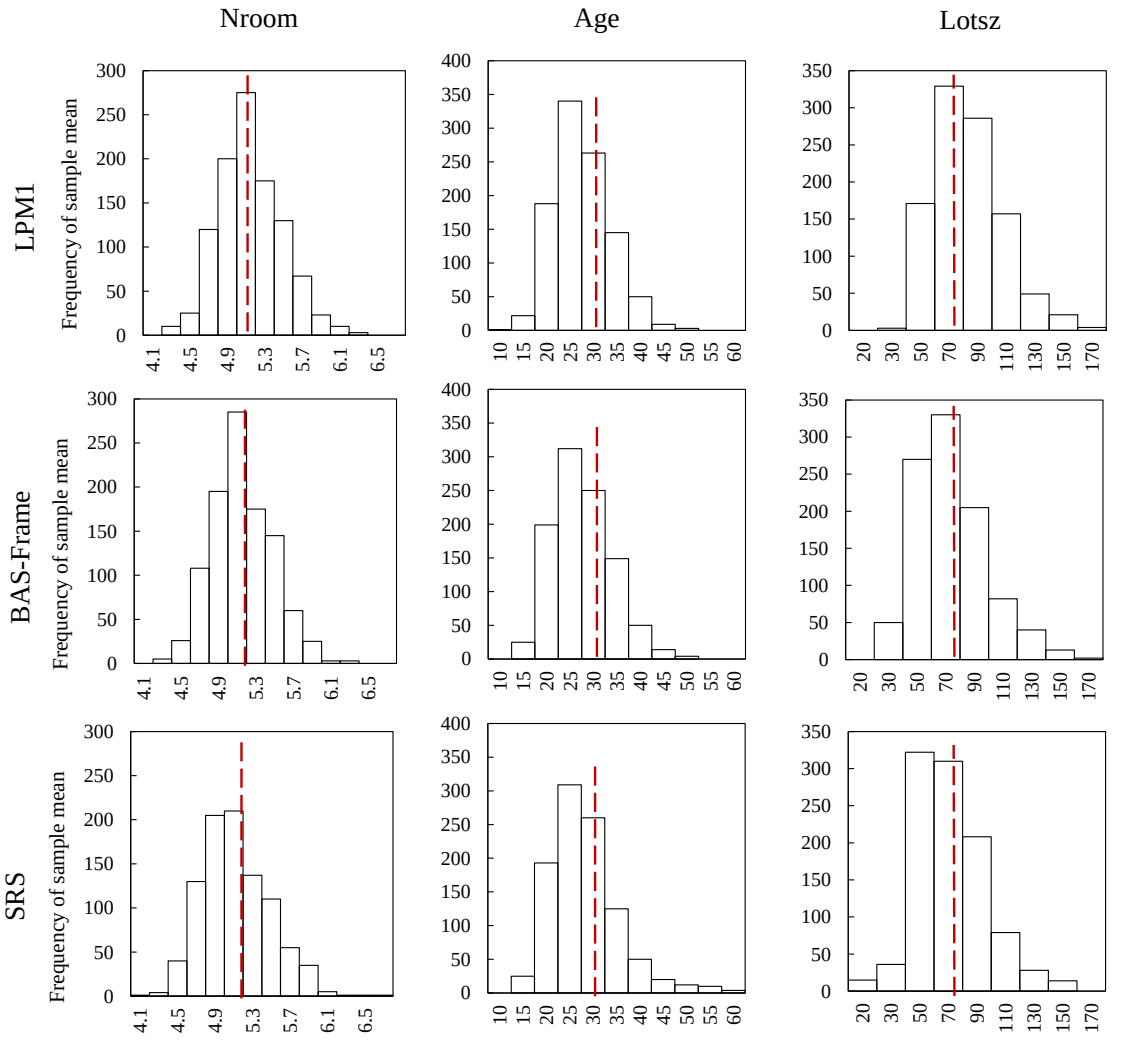


Figure 7-12 Sampling distribution of the auxiliary variables for three different sampling methods among 1000 samples of size 10.

Table 7-4 The simulated variance of the total estimation of the variables of interest where samples are selected by LPM1 and BAS in relation to SRS.

Sample size	Variable	Design	
		$\frac{\hat{V}(\hat{T}_{HT})_{LPM}}{\hat{V}(\hat{T}_{HT})_{SRS}}$	$\frac{\hat{V}(\hat{T}_{HT})_{BAS-Frame}}{\hat{V}(\hat{T}_{HT})_{SRS}}$
10	Price (R)	0.64	0.73
	Nroom (A)	0.85	0.88
	Age (A)	0.84	0.86
	Lotsz (A)	0.80	0.83
15	Price (R)	0.53	0.73
	Nroom (A)	0.75	0.81
	Age (A)	0.85	0.83
	Lotsz (A)	0.79	0.81
20	Price (R)	0.74	0.64
	Nroom (A)	0.86	0.72
	Age (A)	0.88	0.72
	Lotsz (A)	0.85	0.90
25	Price (R)	0.67	0.63
	Nroom (A)	0.85	0.87
	Age(A)	0.83	0.85
	Lotsz (A)	0.89	0.91

Note:
R = Response variable
A = Auxiliary variable

Table 7-4 shows that both LPM and BAS-Frame have smaller simulated variance in estimating the response variable (Price) than SRS when the sampling units are spread not only according to their geographical locations, but also when they are spread over the space of auxiliary variables (two geographical dimensions and three auxiliary variables). Results also showed that spreading the sampling units over the space of the auxiliary variables by using spatially balanced sampling methods provided smaller simulated variance for estimating each auxiliary variable (total of Nroom, Age and Lotsz). Note that, the effect of considering auxiliary variables as stratification variables has been previously investigated in Section 5.4.

7.4.2 Efficiency of BAS-Frame and Number of Auxiliary Variables

There are studies available in the literature that indicate some correlation between points generated in Halton sequences for higher primes (Hess & Polak, 2003; Vandewoestyne & Cools, 2006; Schlier, 2008). For example, the first 10 pairs of points generated by the primes 11 and 13: $(1/11, 1/13), (2/11, 2/13), \dots, (10/11, 10/13)$, have a linear correlation. Helpful displays showing the correlation between dimensions of Halton sequences for higher primes can be found in Chi et al. (2005) and Vandewoestyne and Cools (2006). Correlation between Halton points in higher dimensions may deteriorate the performance of the Halton sequence in generating evenly spread points over an interval. Therefore, it can be concluded that the BAS-Frame method may fail to generate a well-spread sample in the presence of a large number of auxiliary variables. This is shown here through conducting a simulation study on Christchurch meshblocks. In the simulation study, in addition to longitude and latitude of meshblocks, the ten variables listed below were considered as auxiliary variables:

- male: number of males,
- female : number of females,
- Māori: number of Māori,
- child: number of people who are 0 to 14 years old,
- young: number of people who are 15 to 64 years old,
- adult: number of people who are more than 65 years old,
- unemployed: number of unemployed people,
- employed: number of employed people,
- one-storey: number of one-storey housing units,

- one plus storey: number of housing units with more than one storey.

The simulation study was conducted through 10 successive stages in such a way that in each stage a new auxiliary variable was added to the sample selection process. Auxiliary variables were added into the sample selection process in random order. "One plus story" was the only auxiliary variable in the first stage. In the second stage, in addition to "one plus story", "employed" was considered as the second auxiliary variable. The list of auxiliary variables considered in each stage is shown in Table 7-5.

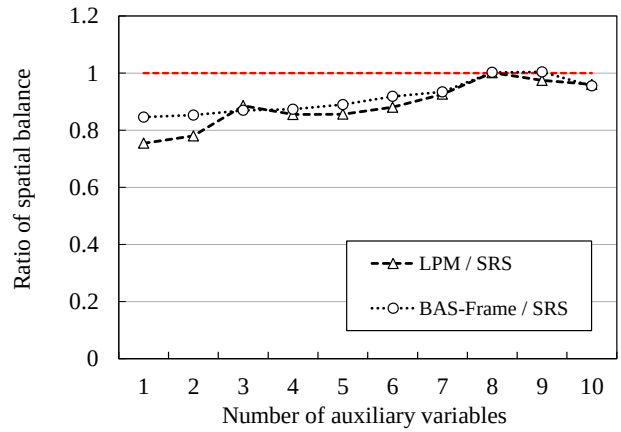
Table 7-5 List of auxiliary variables in each stage of the simulation study.

Stage	Considered auxiliary variables
1	one plus story
2	one plus story, employed
3	one plus story, employed, Māori
4	one plus story, employed, Māori, male
5	one plus story, employed, Māori, male, adult
6	one plus story, employed, Māori, male, adult, one-storey
7	one plus story, employed, Māori, male, adult, one-storey, unemployed
8	one plus story, employed, Māori, male, adult, one-storey, unemployed, child
9	one plus story, employed, Māori, male, adult, one-storey, young, unemployed, child, female
10	one plus story, employed, Māori, male, adult, one-storey, young, unemployed, child, female, young

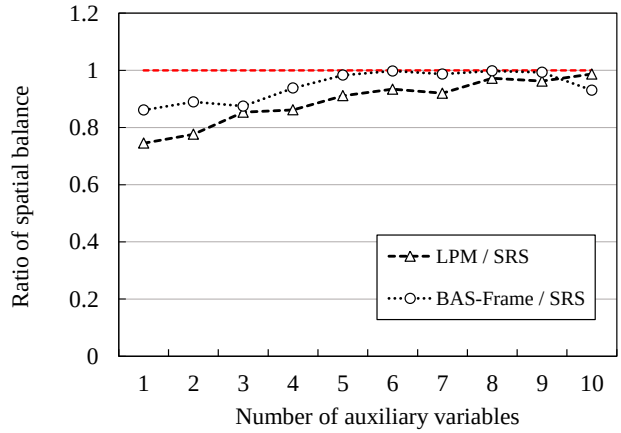
In each stage, 1000 samples were selected using LPM, BAS-Frame and SRS for three different sampling fractions (7%, 9% and 10%). After completing the sample selection process in each stage, the average of spatial balance, ζ , for each sampling method and each sample size was calculated among 1000 iterations. In each stage ζ was calculated (by use of "Balanced Sampling" package in R) according to the geographical coordinates of the meshblocks and distance between the auxiliary variables that were considered in that stage using Equation (7.2). The ratios of the average of ζ for the spatially balanced sampling methods when compared to the relevant values achieved from SRS are illustrated in Table 7-6 and Figure 7-13.

Table 7-6 The ratio of the average of ζ for the spatially balanced sampling methods when compared to the relevant values achieved from SRS, for each sampling fraction and number of considered auxiliary variables.

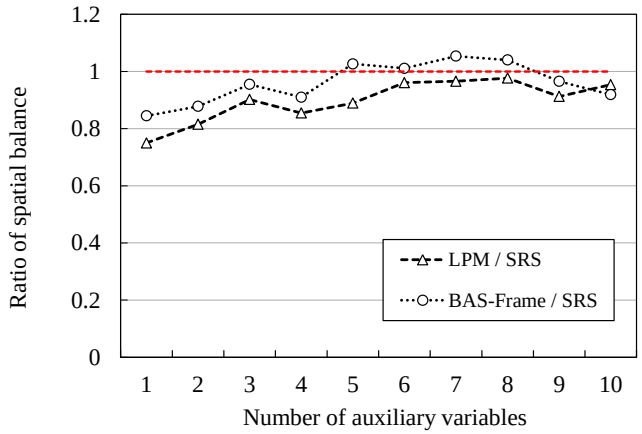
Sampling fraction	Number of auxiliary variables	Sampling design	
		LPM/SRS	BAS-Frame/SRS
7%	1	0.754	0.846
	2	0.780	0.853
	3	0.886	0.870
	4	0.855	0.874
	5	0.856	0.890
	6	0.881	0.918
	7	0.926	0.934
	8	1.002	1.003
	9	0.975	1.004
	10	0.960	0.955
9%	1	0.745	0.861
	2	0.776	0.889
	3	0.853	0.875
	4	0.862	0.938
	5	0.911	0.983
	6	0.934	0.998
	7	0.920	0.987
	8	0.972	0.998
	9	0.962	0.993
	10	0.987	0.930
10%	1	0.750	0.845
	2	0.815	0.878
	3	0.902	0.955
	4	0.855	0.910
	5	0.889	1.026
	6	0.961	1.011
	7	0.966	1.054
	8	0.977	1.040
	9	0.912	0.966
	10	0.954	0.919



(a)



(b)



(c)

Figure 7-13 Trend of average of spatial balance, ζ , for each sampling method amongst the number of auxiliary variables and for a range of sampling fractions: (a) sampling fraction = 7%,(b) sampling fraction = 9% and (c) sampling fraction = 10%.

As Table 7-6 and Figure 7-13 show, for all sampling fractions, for small number of auxiliary variables (when number of auxiliary variables is smaller than 4), both LPM and BAS-Frame methods have a smaller value of ζ, and thus better spatial balance, than SRS. LPM was slightly superior to the BAS-Frame method in terms of spreading sampling units over the population. However, with more auxiliary variables in the sample selection step, the differences between spatial balance from SRS and that from spatially balanced sampling methods decrease. In some cases, the ratio of ζ is greater than 1.

In order to implement BAS-Frame to select well-spread samples when there are a large number of correlated auxiliary variables, there is a need to employ a technique by which the number of auxiliary variables can be reduced. A well-known multivariate technique to reduce a large number of dependent variables to a relatively small set of variables is principal components analysis (PCA: Dunteman, 1989; Jackson, 2005; Jolliffe & Cadima, 2016). PCA provides a set of mutually uncorrelated variables, called principle components (PCs), such that each one is a linear combination of the original variables. The PCs are ordered in a descending trend according to their contribution to total variance, such that the first few PCs represent most of the information in terms of the variation available in the original data. Key results of the PCA can be interpreted through a "scree plot". The scree plot shows the fraction of total variance in the data which are represented by each principle components from largest to smallest.

In this study, PCA was conducted to determine the minimum number of principal components that account for most of the variations in the data of the Christchurch meshblocks. Figure 7-14 shows the "scree plot" for the PCs determined from the Christchurch meshblocks dataset.

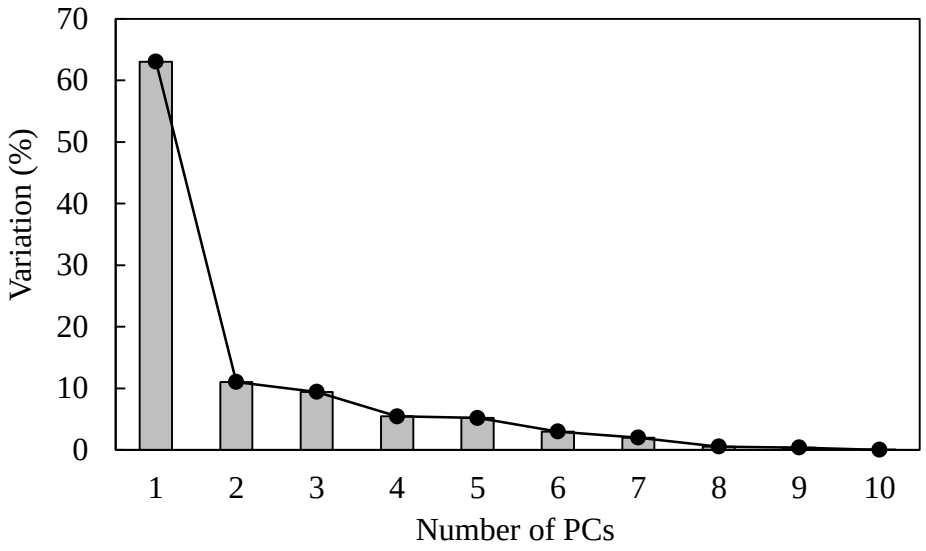


Figure 7-14 The "scree plot" for the PCs determined from the Christchurch meshblocks dataset.

As Figure 7-14 shows, more than 60% of the total variance available in the data can be presented by the first principle component. The first principle component is a linear combination of 10 auxiliary variables considered in this study as follows:

PC1 = 0.34 × (child) + 0.38 × (young) + 0.22 × (adult) + 0.39 × (male) + 0.39 × (female) + 0.27 × (Maori) + 0.37 × (employed) + 0.27 × (unemployed) + 0.30 × (one-storey0 + 0.14 × (one plus storey)

In the next step, a similar simulation study as discussed earlier was carried out on the data of the Christchurch meshblocks. However, in contrast to the previous simulation study, instead of 10 auxiliary variables, only the first principle component (PC1) was considered as an auxiliary variable.

For each sample, after defining Voronoi polygons related to each sampling unit, the ζ explained in Equation (2.22) was calculated as a measure of spatial balance. Note that, Voronoi polygons were defined based on the geographical distances between sampling units and also the distances between units calculated in terms of all auxiliary variables considered in this study. All auxiliary variables were considered for defining Voronoi polygons. This allowed an understanding of how the selected samples are spread over the space of all the auxiliary variables. After completion of the sample selection process, the average of spatial balance, ζ, for each sampling method and each sample size was calculated among 1000

samples. The ratios of the average of ζ for the spatially balanced sampling methods when compared to the relevant values achieved from SRS are illustrated in Table 7-7.

Table 7-7 The ratio of the average of ζ for the spatially balanced sampling methods when compared to the relevant values achieved from SRS.

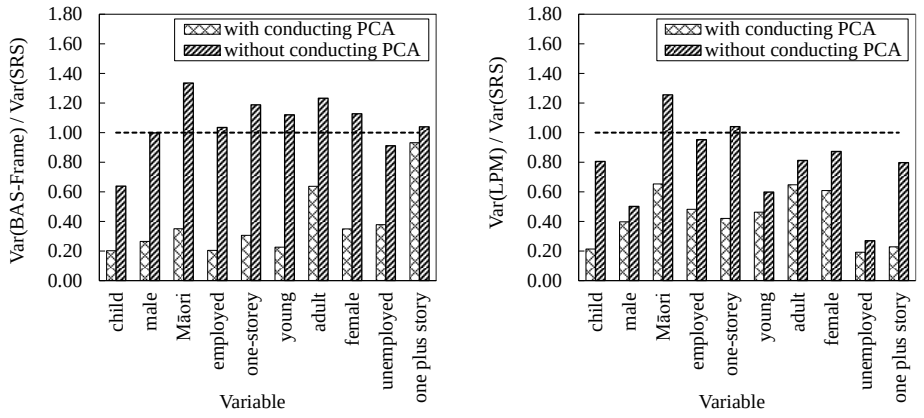
Sampling fraction	Sampling design	
	LPM/SRS	BAS-Frame/SRS
7%	0.805	0.833
9%	0.833	0.893
10%	0.784	0.837

As Table 7-7 illustrates, all the achieved ratios are smaller than 1. This shows that, spatially balanced sampling methods when the first principle component was considered as the only auxiliary variable, provided more spatially balanced samples than SRS. Comparing the results which are reported in Table 7-7 with the relevant values (values corresponding to situations that all 10 auxiliary variables were considered in the sample selection process) in Table 7-6 shows that considering PC1 instead of a list of all auxiliary variables provided more spatially balanced sample in the both BAS-Frame and LPM.

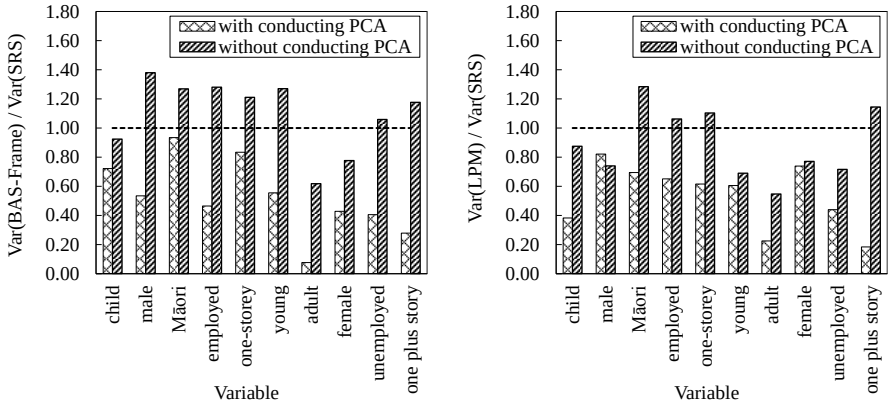
To investigate how the consideration of PCA during sample selection process can increase the precision of estimates, simulated variances of the mean estimation of the auxiliary variables were compared in two situations: (1) when PC1 was considered as the only auxiliary variable in the sample selection process, and (2) when all 10 auxiliary variables were considered in the sample selection process. The simulated variances of the auxiliary variables for LPM and BAS-Frame in relation to SRS for three sampling fractions and two situations are reported in Table 7-8 and Figure 7-15.

Table 7-8 The simulated variances of the auxiliary variables for LPM and BAS-Frame in relation to SRS for three sampling fractions (7%, 9% and 10%) and two situations: (1) when PC1 was the only auxiliary variable in the sample selection process, and (2) when all 10 auxiliary variables were considered in the sample selection process.

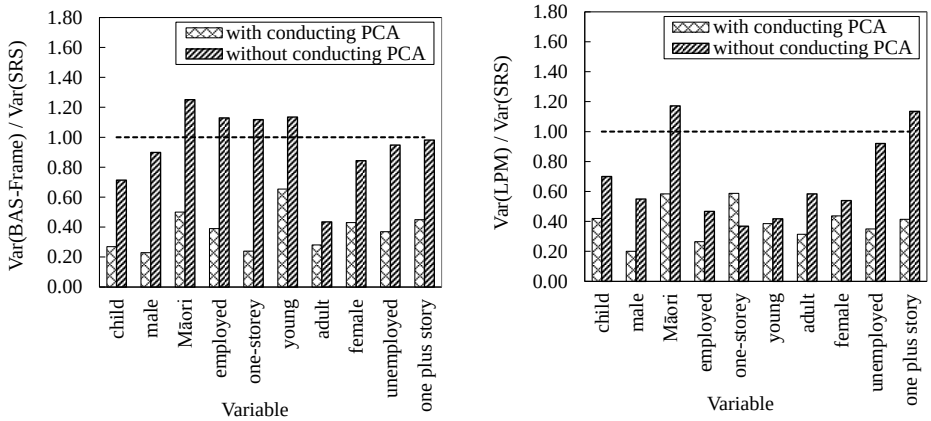
Sampling fraction	auxiliary variables	Sampling design			
		BAS-Frame/SRS		LPM/SRS	
		With conducting PCA	without conducting PCA	With conducting PCA	without conducting PCA
7%	child	0.20	0.64	0.21	0.80
	male	0.26	1.00	0.40	0.50
	Māori	0.35	1.34	0.65	1.25
	employed	0.20	1.03	0.48	0.95
	one-storey	0.30	1.19	0.42	1.04
	young	0.22	1.12	0.46	0.60
	adult	0.64	1.23	0.65	0.81
	female	0.35	1.13	0.61	0.87
	unemployed	0.38	0.91	0.19	0.27
	one plus story	0.93	1.04	0.23	0.80
9%	child	0.72	0.92	0.38	0.87
	male	0.53	1.38	0.82	0.74
	Māori	0.93	1.27	0.69	1.28
	employed	0.46	1.28	0.65	1.06
	one-storey	0.83	1.21	0.61	1.10
	young	0.55	1.27	0.60	0.69
	adult	0.27	0.62	0.22	0.55
	female	0.43	0.78	0.74	0.77
	unemployed	0.40	1.06	0.44	0.72
	one plus story	0.28	1.18	0.18	1.14
10%	child	0.27	0.71	0.42	0.70
	male	0.23	0.90	0.20	0.55
	Māori	0.50	1.25	0.58	1.17
	employed	0.39	1.13	0.26	0.47
	one-storey	0.24	1.12	0.59	0.37
	young	0.65	1.14	0.39	0.42
	adult	0.28	0.43	0.31	0.58
	female	0.43	0.84	0.44	0.54
	unemployed	0.37	0.95	0.35	0.92
	one plus story	0.45	0.98	0.41	1.14



(a)



(b)



(c)

Figure 7-15 The simulated variances of the auxiliary variables for LPM and BAS-Frame in relation to SRS for two situations: (1) when PC1 was the only auxiliary variable in the sample selection process, and (2) when all 10 auxiliary variables were considered in the sample selection process and for three sampling fractions: (a) sampling fraction = 7%,(b) sampling fraction = 9% and (c) sampling fraction = 10%.

As Table 7-8 and Figure 7-15 show for all the auxiliary variables, the ratios of the simulated variances for both LPM and BAS-Frame in comparison to SRS are less than 1 when PC1 was considered as the only auxiliary variable. When PCA is included in the sample selection process, these ratios are smaller than ratios corresponding to the situation that PCA was not considered. Results confirmed that considering PC1 as the only auxiliary variable in the sample selection process provided smaller simulated variance for estimating each of the auxiliary variables.

The studies presented in this section showed that BAS-Frame can work as well as LPM in terms of spreading the samples over the geographical region of the population and the space of the auxiliary variables. The first principle component (PC1) was considered as the only auxiliary variable because it explained more than 60% of the total variance available in the data. However, in general, the number of required PCs is likely to vary in different situations and should be determined using the "scree" plot.

7.5 Conclusions

The feasibility of applying spatially balanced sampling methods for dealing with some common features of household sampling surveys was investigated in this chapter.

Combining undersized units in order to define PSUs with desirable sizes is one of the features of household surveys studied in the first section. A famous method recommended by the United Nations for constructing PSUs in developing country is the Kish method. Although this method is easy to implement, it does not guarantee that the created PSUs include the neighbouring units. In this chapter, a new technique for combining undersized units based on the rationale of the BAS-Frame method was introduced. The performance of this technique in terms of combining nearby units to form a PSU with a desirable size was compared with the performance of the Kish method through running a simulation study. Results of the simulation study showed that for all considered thresholds for defining PSUs, the average of distances between combined units (calculated using the travelling salesman problem) in the new technique was shorter than the distances between units combined together by the Kish method.

Available literatures show that BAS and GRTS are able to add new sampling units to the current sample, while keeping the spatial balance (Stevens, D. & Olsen, 2004; Robertson Robertson et al., 2013). This study showed that such beneficial characteristic can also be

accomplished using BAS-Frame. This feature of the BAS-Frame method makes it suitable for longitudinal designs. It is likely that the same households or PSUs would be selected in different household surveys when BAS-Frame is employed; however, as discussed, by increasing the size of the population the overlap between successive surveys is expected to be decreased.

The application of the BAS-Frame method in the presence of auxiliary variables was studied in the last section. The study showed that when there are a small number of auxiliary variables, the BAS-Frame method is able to spread the sampling units not only over the geographical space of the population, but also over the space of the auxiliary variables. However, its performance in spreading a sample over the space of the auxiliary variables decreases as the number of auxiliary variables is increased. Principle component analysis was used to reduce the number of correlated auxiliary variables.

After defining the required number of principle components (PCs) that explain an acceptable percentage of the variation in the data (in this study 60% of the variation of the data was explained by PC1), BAS-Frame method was employed to select samples according to the selected PCs instead of considering all the auxiliary variables. The results showed that the selected samples were more spatially balanced than the situation where all the auxiliary variables were considered in the sample selection process.

7.6 References

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Chapter 8 *Conclusions and Recommendations*

The structure and characteristics of the target population determine the choice of sampling design. In household surveys – often a country's main tool for collecting socio-economic information – the target populations usually consist of households or individuals who are living in a specific geographical region and have interactions with their neighbours. Design units (i.e., households, or individuals) that are nearby are more similar than units farther away. Given the existence of spatial correlation in the population, selecting sampling units close to one another may provide similar information in the sample, and this may have negative effects on the precision of sample estimates.

Spatially balanced sampling has been introduced to increase the efficiency of samples in providing more information per sample unit by maximizing spatial independence among sampling units (Theobald et al., 2007). Although the methods of spatially balanced sampling have been used in sampling natural resources and environmental phenomena, there are few studies in the literature where these methods are applied in socio-economic studies.

This thesis investigated the idea of using spatially balanced sampling methods in selecting samples for household surveys. The overall goal of this thesis was to understand the potential application of spatially balanced sampling in household surveys to select more representative samples.

This thesis is mostly focused on using, and extending, balanced acceptance sampling (BAS; Robertson et al., 2013). One reason for the focus on this design is because BAS is based on a relatively simple algorithm, and can easily be used in a large population.

In order to address the main objectives of the research, the study was conducted in two parts: in the first part, comprising Chapters 3 and 4, the practical aspects of the BAS method were investigated for application to environmental studies. In the second part, Chapters 5, 6, and 7, spatially balanced sampling was applied to household surveys.

8.1 Key Contributions

8.1.1 Part 1: Practical Aspects of the BAS Method

The first part of this study attempted to provide a more detailed investigation of the advantages of using the BAS method.

To this end, in Chapter 3, the BAS method was employed to select a spatially balanced sample from the crab population in Alkhor, on the east coast of Qatar. This chapter compared the BAS method with the two-dimensional systematic sampling, a popular sampling method used in environmental studies.

Results achieved from applying the BAS and the two-dimensional systematic method on the crab population showed that these two designs had almost similar efficiency in selecting spatially balanced samples and estimating the parameter of interest (i.e., size of the crab population). However, there are some practical advantages with BAS that make it superior to the two-dimensional systematic sampling:

- In two-dimensional systematic sampling, the coverage of the study area would only be met once the sampling process is completed, while BAS is able to cover the study area even when the sampling process is stopped early.
- With BAS, new sampling units can easily be added to the sampling process in order to compensate for missing values, whereas adding new sampling units with the two-dimensional systematic sampling may result in a loss of the spatial balance.
- BAS can be used to select unequal probability samples.
- BAS can be used for selecting any sample size, while systematic sampling cannot be used easily (without modification) for selecting a sample for example, when the sample size is a prime number.

In Chapter 4, the performance of the BAS method was evaluated for two different types of populations (i.e., a population where the observations followed a Gaussian distribution, and when the observations are binary) and with different levels of spatial autocorrelation. Results of the simulation studies in this chapter showed that despite considering different types of populations, by increasing the spatial autocorrelation (which is measured by Moran's I in this thesis), the precision of the population estimates increased compared with the estimates achieved from simple random sampling (SRS) when the BAS method was used.

This chapter also assessed the suitability of applying BAS in stratified populations without it being necessary to stratify the population explicitly. The simulation study on the crab population showed that the BAS method can be used as an alternative to stratified sampling with proportional allocation. In this situation, there is no need to create explicit strata. In Chapter 4, it was also shown that when selection of samples with different sampling fractions in strata was required, the BAS method can be used within each stratum independently (in other words an independent BAS sample can be taken from each stratum, rather than one BAS sample with different within-stratum sample intensities).

The finding from the first part of the thesis suggested that, in general, the BAS method as a spatially balanced sampling method has potential to be used for selecting samples in a range of practical settings and has promising potential for it to be extended for application in other surveys such as household surveys.

8.1.2 Part 2: Application of Spatially Balanced Sampling in Household Surveys

The main goal of the second part of this thesis was to find out if a possibility exists for improving the precision of sampling designs of household surveys by applying spatially balanced sampling methods.

To answer this question, this thesis firstly provided a clear description of dissimilarities between objectives and target populations in environmental studies and household studies. Then it introduced some new techniques for applying the spatially balanced sampling in household surveys. Ultimately, the benefits of the application of spatially balanced sampling in household surveys were highlighted.

While samples in environmental studies are usually selected from continuous populations, the study populations in household surveys typically consist of irregular finite discrete units. In this case, implementing the BAS method for selecting spatially balanced samples may not be helpful (in that it may select samples that are not spread evenly over the population). To address this in the BAS method, to select spatially balanced samples from irregular discrete populations, a new modification of the BAS method, the BAS-Frame method, was developed, as described in Chapter 5. The spatial and statistical properties of the proposed method were investigated through conducting simulation studies. The results of the simulation studies indicated that the BAS-Frame method is able to select spatially balanced samples as well as other spatially balanced sampling methods.

As described in Chapter 5, to implement BAS-Frame for selection of spatially balanced samples, there is a need to initially create a spatial frame of the population of interest. For selecting samples with equal inclusion probability, the creation of the spatial frame can be achieved by either adding points to, or removing points from, the population randomly. The simulation studies showed that the application of the BAS-Frame technique leads to more spatially balanced samples when the random points are removed. However, for selecting samples with unequal inclusion probability, random points could not be removed randomly as the population units may not have the same probability of selection. In these cases, the primary frame is recommended to be created by adding random points that have selection probability equal to zero.

Chapter 5 also studied the application in real situations of the BAS-Frame and of other spatially balanced sampling methods available in the literature. For this, the spatially balanced sampling methods were compared to each other, in terms of spreading sampling units over the population and providing more precise estimates, through conducting a simulation study for selecting samples from a list of meshblocks in the region of Canterbury, New Zealand. The results of the simulation study showed that, while there was a relatively poor spatial autocorrelation for the considered response variables, the spatially balanced sampling methods provided more precise estimates when compared with the SRS method. This means that using spatially balanced sampling methods can improve the precision of estimates in household surveys. Among the sampling methods considered in this study, the local pivotal method (LPM) had the best performance in terms of spreading the sampling units over the population and providing smaller $Deff$.

In order to increase the precision of estimates and ensure that important groups in the population have proper representation in the sample, target populations in household surveys are usually stratified by either geographic or demographic characteristics of the units. In this thesis, the application of the BAS-Frame method in selecting spatially balanced samples from discrete stratified populations was evaluated. Two kinds of populations were considered: a population that was stratified according to geographical characteristics (i.e., urban or rural), and a population that was stratified based on demographic characteristics (i.e., ethnicity).

Regarding the use of geographical characteristics in the stratification process, target populations in household surveys are usually stratified into urban and rural strata. This stratification is usually used to control the survey costs in each stratum. As travelling costs in

rural areas are generally higher than travelling costs in urban areas, selecting a spatially balanced sample might not be preferable in rural areas. To investigate travel costs when the BAS-Frame method is used for rural areas, in comparison to SRS, a simulation study on meshblocks of Ashburton, a town in New Zealand, was conducted. Results of the simulation study confirmed that using spatially balanced sampling in a rural stratum cost more than using the SRS method. However, travel cost increase was not marked in the urban stratum. According to these results, for household surveys, applying spatially balanced sampling in urban areas might be more desirable than its application in rural areas.

In addition to geographical stratifications, target populations in household surveys may be stratified by socio-economic and demographic characteristics of units. There can be some difficulties in applying the stratified sampling method in household surveys, such as selecting the relevant auxiliary variables. This motivated us to investigate whether the stratified sampling method in household sampling designs can be replaced with spatially balanced sampling. Chapter 5 of the thesis focused on two situations: when the stratification is used only to ensure that all groups in the population, especially groups which make up a small proportion of the population, are present in the sample; and when the survey is a multi-objective survey that aims to estimate several population characteristics within a single survey.

To investigate the effect of spatially balanced sampling in the first situation, a simulation study was undertaken on meshblocks of Christchurch city. The simulation study compared the performance of spatially balanced sampling methods with proportional stratified sampling for selecting samples with proper representation of Māori. Findings of the simulation study showed that the results from using the spatially balanced sampling methods satisfactorily matched the results achieved from the proportional stratified sampling method. This confirmed that implementing spatially balanced sampling on irregular discrete populations in household surveys can perform as well as proportional stratified sampling. Therefore, in cases where the same sampling fraction is used in each stratum, a BAS-Frame method can be recommended as an alternative method to proportional stratified sampling.

In the second situation, we investigated the application of spatially balanced sampling in multi-objective surveys. Finding a suitable stratification variable that is related to all the characteristics under study is one of the important, and more difficult, tasks in using a stratified sampling in multi-objective surveys. One suggestion to overcome the difficulty of

selecting a proper stratification variable is to use a spatially balanced sampling method instead of a stratified sampling method. This idea was investigated in Chapter 5 by conducting a simulation study on meshblocks of Christchurch city. In the simulation study, only one out of five target variables considered in a multi-objective survey seemed to have a correlation with the stratification variable. One finding of the simulation study was that, by using the stratification technique, a more precise result was achieved in estimating the target variable that had a correlation with the stratification variable. In contrast, for other target variables, without a correlation with the stratification variable, the spatially balanced sampling methods provided more precise estimates than the stratified sampling method. This result suggested that in the use of spatially balanced sampling in household surveys where the aim is to estimate a number of target variables, finding the relevant stratification variables may not be possible.

Chapter 6 investigated the feasibility of applying spatially balanced sampling in the presence of different sampling frames in household surveys. The application of the spatially balanced sampling methods in the presence of area frames and list frames – the two main kinds of sampling frames in household surveys – was studied in the first part. In this study, the spatially balanced sampling methods were compared with conventional sampling techniques (simple random sampling, systematic sampling, proportional to size sampling) in a two-stage cluster sampling method that is used in household surveys. The results achieved in this part indicated that the current sampling methods used in most household surveys can be substituted with spatially balanced sampling methods.

In the second part of Chapter 6, the implementation of spatially balanced sampling in the presence of a list of registered addresses, which is a new form of a sampling frame in household surveys, was investigated. While employing spatially balanced sampling methods for selecting samples from a list of registered addresses can provide more precise estimates, it may increase the survey cost when a face-to-face interview is required for collecting data. To overcome this problem, in this part, a new modification of the BAS-Frame method – called Cluster BAS-Frame method – was introduced. The Cluster BAS-Frame method has the same rationale as the BAS-method and is used to control the survey cost by putting nearby units in the same cluster. The efficiency of the Cluster BAS-Frame method was investigated by conducting a simulation study on an artificial population generated from the Christchurch City meshblocks information. The results of the simulation study confirmed that using the

Cluster BAS-Frame method can decrease the survey costs when compared with applying the BAS-Frame method for selecting a spatially balanced sample.

Chapter 6 also studied the application of spatially balanced sampling methods in non-ideal situations where an ordinary sampling frame (i.e., area frame and list frame) is not available. In this situation, after providing a map of the population of interest and defining the geographical centre of areas, the available spatially balanced sampling methods can be applied to select sample units. This chapter also discussed situations where statisticians prefer to select sample areas based on their geographical boundaries. Among the spatially balanced sampling methods available in the literature, BAS can be used for selecting a spatially balanced sample from a map-based frame where units are defined only by their boundaries. In this situation, an extra dimension is introduced, the inverse of the area of each unit, and then samples are selected by implementing an acceptance/rejection technique. The efficiency of the application of BAS in this situation was compared with the efficiency of the BAS-Frame method and a modified version of SRS in terms of selecting spatially balanced sample areas from a map by conducting a simulation study. The results achieved from the simulation study showed that the BAS method selected samples had higher spatial balanced compared to the modified version of SRS. In addition, it was found that the implementation of the BAS-Frame method on the basis of the centres of the areas provided more balanced samples compared with the situation that sample areas were selected from the map using the BAS method.

The applicability of the spatially balanced sampling methods to deal with some features of designing a household survey was demonstrated in Chapter 7. Defining primary sampling units (PSUs) which meet a pre-specified minimum number of sampling units, the desirable size of the PSU, is one of the important aspects of designing household surveys. In the first part of Chapter 7, a new technique based on the rationale of the BAS-Frame method was introduced to construct PSUs with desirable sizes. This method tries to merge undersized units with their nearby units as much as possible. A simulation study was undertaken to compare the performance of the proposed method with the Kish method, which is a method recommended by the United Nations for constructing PSUs in developing countries. The results of the simulation study showed that the new method combined more nearby units than the Kish method.

Household surveys often implement a longitudinal design to monitor the changes in parameters of interest over a time period. As discussed in Chapter 7, BAS-Frame (similar to BAS and GRTS) is able to add new sampling units to the current sample, while keeping the spatial balance. This characteristic makes it suitable for implementing in the longitudinal designs. In fact, using the BAS-Frame method can allocate sampling units into the rotation groups, such that there is no overlap between rotation groups and selecting neighbors in a same rotation group is avoided. This is done by discarding Halton points associated with the sampling units selected in several rotation groups.

The application of BAS-Frame for selecting independent household surveys may result in the selection of the same sampling units (i.e., PSUs or households) in different household surveys; however, such overlap between samples is expected to be decreased by increasing the size of the population.

Sampling frames in household surveys usually include a number of auxiliary variables that can be used in designing a suitable sample for household surveys. Chapter 7 discussed how spatially balanced sampling methods could incorporate information from auxiliary variables on selecting representative samples. The simulation studies undertaken in this chapter showed that when there were a small number of auxiliary variables, the BAS-Frame method was able to spread the sampling units not only over the geographical space of the population, but also over the space of the auxiliary variables. However, in cases with more auxiliary variables, the performance of the BAS-Frame method in spreading a sample over the space of auxiliary variables was rarely better than SRS.

In order to select spatially balanced samples in this situation when there were a large number of correlated auxiliary variables, principle component (PC) analysis was firstly conducted to reduce the number of auxiliary variables, and then the BAS-Frame method was employed on the selected PCs instead of considering all the auxiliary variables. In this example, the first principle component (PC1) was considered as the only auxiliary variable as it explained more than 60% of the total variance available in the data. The results of the study indicated that considering PCs in the sample selection process provided more spatially balanced sample than situations in which all the auxiliary variables were considered in the sample selection process. The use of PC analysis in the sample selection process also resulted in a smaller simulated variance for estimating each of the considered auxiliary variables.

8.2 Recommendations for Future Work

This thesis has illustrated the advantages of employing spatially balanced sampling in household surveys by selecting more representative samples. Considering the limitations of the existing literature, the research presented in this dissertation can be extended in multiple ways. The following recommendations are made for future work:

- In this thesis, it was assumed that nearby area units (e.g., meshblocks) in household surveys tend to be more similar than units farther apart. However, in reality, one may need to consider more geographical features to define similar area units. For example, when considering trade or a spread of epidemics, adjacent area units divided by a river or a mountain range will not be considered neighbours, while regions geographically distant from each other, but connected by a high speed railway will be. Some spatially balanced sampling methods (e.g., LPMs, SCPS) can deal with this problem by manipulating the matrix of spatial weights that shows the distance between different areas. However, further research is needed to modify the BAS-Frame method in such a way as to address this practical requirement.
- Due to lack of information, in some simulation studies undertaken in this thesis, it was assumed that the survey cost is affected only by travelling distance, i.e., the length of the path among selected units. These simulation studies could be repeated again including more practical covariates. For example, adding interviewer's wages, fuel cost and other factors to model a survey cost.
- A practical modification of the BAS-Frame method (called Cluster BAS-Frame) for selecting a sample from a list of registered addresses, which is a new form of sampling frame, was introduced in Chapter 6. Defining the size of clusters is a challenging task for this method and can be affected by spatial autocorrelation of units within each cluster. A possible study can be conducted to determine suitable cluster sizes based on the spatial characteristics of the population units and the estimated budget available for conducting the survey.
- The advantages of using the BAS-Frame method to create rotation groups were discussed in Chapter 7. Further work is recommended to provide suitable estimators for estimating the parameters of interest (e.g., mean, total) in each rotation group.

- This thesis studied the possibility of spatially balanced sampling in selecting a representative sample for household surveys. Possible future studies can include investigating the effect of applying spatially balanced sampling on other parts of designing a household survey (e.g., imputing missing values and estimating variance of parameters of interest).

8.3 References

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Appendix A An Algorithm for Simulating a Spatial Auto-Correlated Population

A.1 Introduction

In most sampling surveys studies, the statistical efficiency of a newly introduced method is demonstrated and assessed by comparison with at least one population data set. However, because of, for example, privacy laws and high costs, access to real population data is often very limited and, instead, synthetic populations are used. The important feature in creating a synthetic population is its similarity to the real population data, while at the same time it must ensure confidentiality (Alfons et al., 2010). The need to create reliable synthetic populations has attracted attention from a number of researchers (see Müller et al., 2010; Barthelemy & Toint, 2013), particularly the desire to generate synthetic population data with a pre-specified level of spatial auto-correlation. A number of methods have been developed to achieve this goal in various fields, including climate science (Wilks, 1999; Sharif & Burn, 2006; Efstratiadis et al., 2014; King et al., 2014), archaeology (Orton, 1982), and urban and demographic policy (Ballas et al., 2007; Smith, D. et al., 2009; Barthelemy & Toint, 2013).

In the context of demographic studies, and indeed many other contexts, there may also be interest in simulating a spatial auto-correlated population when the response variable(s) is binary or dichotomous. Examples include gender (male/female), employment status (employed/unemployed), whether or not there is a child in the household, availability of the internet in the household, etc. Dichotomous variables can also occur in other fields of study. For example, a geographic area may be declared a pest-free zone or not.

A.2 Generating a Population With a Specified Spatial Auto-Correlation

A random vector x of observations from a given distribution (i.e. Gaussian distribution) could be generated by employing one of a number of available sampling methods to select units from the distribution of interest. However, if there is a level of spatial interaction in the real population then in the simulated population the random values should be arranged to reflect the specific spatial autocorrelation. One possible way to do this is to examine all possible arrangements of the generated units to find the suitable arrangement with the desired spatial

autocorrelation. As an another option, one may use the algorithm introduced by Goodchild and Openshaw (1980) which is based on Moran's I.

As was explained in Chapter 2, Moran's I is an index to reflect spatial codependency or autocorrelation between the values of the variables of interest in the neighboring units located in a region. Although, Moran's I has a long history of being used to express the degree of spatial autocorrelation, it has rarely been used in generating auto-correlated synthetic populations. Goodchild and Openshaw (1980) introduced an algorithm for producing a random field consists of N observations from a Gaussian distribution so that the generated observations yield a desired Moran's I (denoted as I^*). The algorithm of generating a spatial auto-correlated population with a normal response variable ($x \sim N(\mu_x, \sigma_x)$) proceeds as follows.

Let n^* and ε be the maximum allowed number of iterations and the value of a considered tolerance, respectively . The tolerance ε implies that any suggested Moran's I such that $|I - I^*| < \varepsilon$ will be acceptable. For example, if we want to achieve $I^* = 0.5$ with tolerance $\varepsilon = 0.01$, then any value of I between 0.49 and 0.51 will be acceptable.

To initiate, on a given spatial structure (for simplicity, we suppose that the region of interest has a regular rectangular shape), generate an initial vector of values $x_i, i = 1, \dots, N$ from the pre-specified Gaussian distribution with mean μ_x and standard deviation σ_x. Calculate Moran's I for the current configuration. Then, choose two zones, k and m, at random and propose to swap the values x_k and x_m in them. If the swap brings the Moran's I closer to the desired value, accept the swap. Otherwise, propose a different swap. Continue, until the current Moran's I is close enough to the desired value or until the maximum number of iterations has been reached.

Goodchild's algorithm for generating a random standard Normal field with a desired Moran's I is summarized in Figure A-1.

Define desired Moran's I (I^*)
Define tolerance ε
Define maximum allowed number of iterations (n^*)

STEP 1:
on a given spatial structure, generate a list of values ($\{x_i, i = 1, \ldots, N\}$) from standard Normal distribution.

STEP 2:
calculate Moran's I for the current generated values (I).

While $|I - I^*| > \varepsilon$ and $n < n^*$ **Do**

STEP 3:
randomly select a pair of zones k and m,
swap their values ($x_m \leftrightarrow x_k$)
compute the new Moran's I for this new field (I_{new}).

STEP 4:
IF $|I_{new} - I^*| < |I - I^*|$,
THEN accept the swap,
ELSE continue.

Figure A-1 Goodchild's algorithm to generate a spatial auto-correlated population from standard Normal distribution.

A.2.1 Generating a Spatially Auto-Correlated Bernoulli Population

As mentioned before, there are situations (especially in demographic studies) that the variable of interest is binary rather than Gaussian. In this case $x_i \in \{0,1\}$ for $i = 1, \ldots, N$ and $\Pr(x_i = 1) = 1 - \Pr(x_i = 0) = p$. To generate a random spatial auto-correlated population from a Bernoulli distribution with parameter p, one can easily implement the Goodchild's algorithm. For this, instead of Normal distribution, the list of observations should be randomly selected from Bernoulli distribution with probability p. The Goodchild's algorithm for generating a spatially auto-correlated Bernoulli population with probability p is summarized in Figure A-2.

Define desired Moran's I (I^*)
Define tolerance ε
Define maximum allowed number of iterations (n^*)

STEP 1:
on a given spatial structure, generate a list of values ($\{x_i, i = 1, \ldots, N\}$) from Bernoulli distribution with probability p.

STEP 2:
calculate Moran's I for the current generated values (I).

While $|I - I^*| > \varepsilon$ and $n < n^*$ **Do**

STEP 3:
randomly select a pair of zones k and m,
swap their values ($x_m \leftrightarrow x_k$)
compute the new Moran's I for this new field (I_{new}).

STEP 4:
IF $|I_{new} - I^*| < |I - I^*|$,
THEN accept the swap,
ELSE continue.

Figure A-2 Goodchild's algorithm to generate a spatial auto-correlated population from Bernoulli distribution with probability p.

In the above algorithm (Figure A-2), since, obviously, if $x_m = x_k$ (when both zones have similar value equals 1 or 0), the swap makes no change to the Moran's I. Therefore, a considerable amount of time is wasted just to swap the units by similar values. This thesis, to address this, suggests checking for such an event before proceeding.

In fact, the efficiency of the algorithm can be improved by simply making sure that, for example, index k is always sampled from the indices of zones with $x = 0$ and index m is always sampled from those of zones with $x = 1$. The modified algorithm which is proposed in this thesis is summarized in Figure A-3.

Define desired Moran's I (I^*)
Define tolerance ε
Define maximum allowed number of iterations (n^*)

STEP 1:
on a given spatial structure, generate a list of values ($\{x_i, i = 1, \ldots, N\}$) from Bernoulli distribution with probability p.

STEP 2:
calculate Moran's I for the current generated values (I).

While $|I - I^*| > \varepsilon$ and $n < n^*$ **Do**

STEP 3:
select a zone which has value 1 and a zone with value 0 randomly $(k, m)\ s.t.\ x_k = 0, x_m = 1$,
swap their values $(x_m \leftrightarrow x_k)$
compute the new Moran's I for this new field (I_{new}).

STEP 4:
IF $|I_{new} - I^*| < |I - I^*|$,
THEN accept the swap,
ELSE continue.

Figure A-3 The modified Goodchild's algorithm to generate a spatial auto-correlated population from Bernoulli distribution with probability p. In this algorithm zones with different values are selected to swap.

Figure A-4 presents the result of a single application of the modified Goodchild's algorithm on a random Bernoulli population of size 10×10 with $p = 0.5$. At the beginning of the algorithm, units in the population are arranged randomly (

Figure A-4a), therefore the spatial auto-correlation between their values is $I = -0.02$. In Figure A-4, units which have the characteristic of interest ($x_i = 1$) are shown by black solid circles.

Figure A-4b illustrates the resultant population distribution after running the algorithm. The pattern now is a spatially auto-correlated Bernoulli population with the desired Moran's I of $I^* = 0.4$ when tolerance $\epsilon = 0.01$ is considered. Here, after 100 iterations, the Moran's I has increased from $I = -0.02$ to $I = 0.39$.

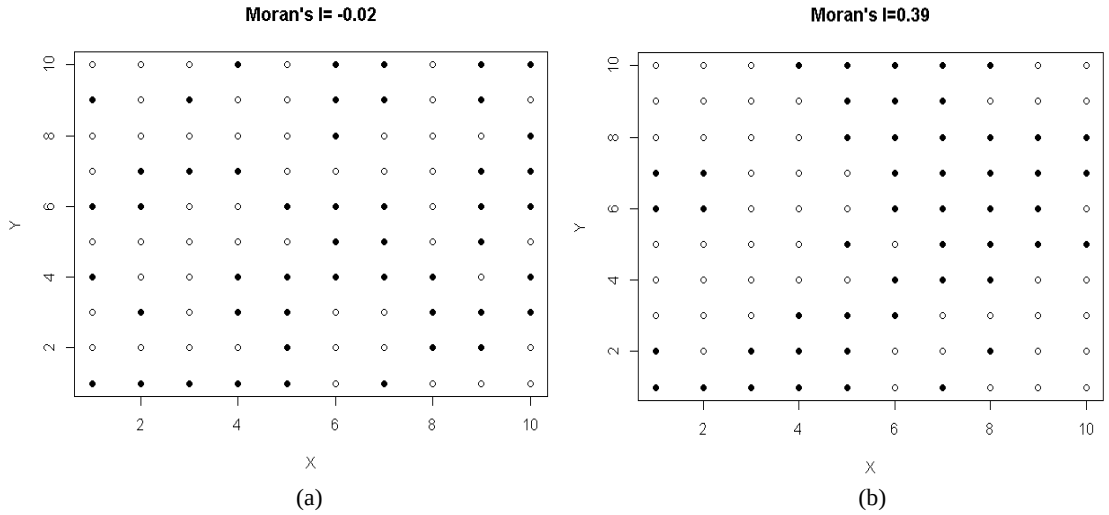

Figure A-4 a) a randomly generated population from Bernoulli distribution with p=0.5. Units with the characteristics of interest are shown by solid black circle. The spatial auto-correlation between values of the units is I=-0.02. b) the synthetic auto correlated population achieved after applying the modified Goodchild's algorithm. After 100 iterations, the Moran's I of I=0.39 was achieved.

The Moran's I achieved in each repeat of applying the modified Goodchild's algorithm to the above population is shown in Figure A-5. In Figure A-5 there is a clear trend of increasing the value of Moran's I as the number of iterations is increased.

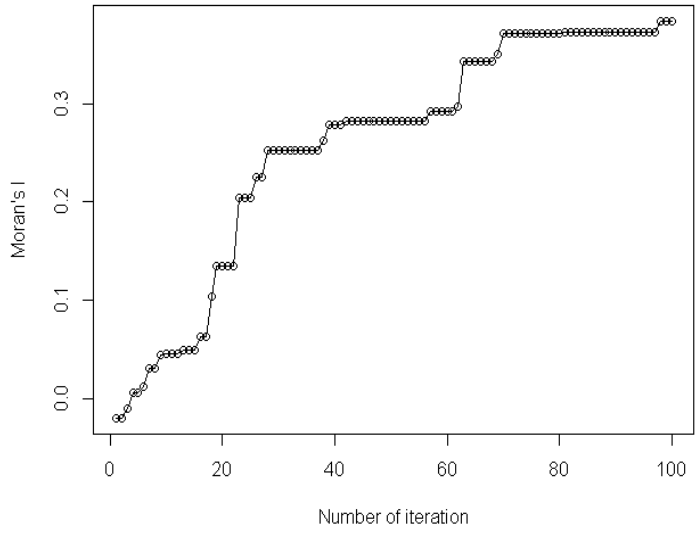

Figure A-5 The Moran's I achieved in 100 first iteration of applying the modified Goodchild's algorithm to a randomly created population from a Bernoulli distribution with p=0.5. The Moran's I has changed from I=-0.02 to I=0.39.

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